

P L A I N TRIGONOMETRY

Rendered *Easy* and *Familiar*,

By CALCULATIONS in ARITHMETICK only:

WITH ITS

APPLICATION and Use

In ascertaining all Kinds of HEIGHTS, DEPTHS, and DISTANCES,

IN THE

HEAVENS, as well as on the EARTH and SEAS;

WHETHER OF

Towers, Forts, Trees, Pyramids, Columns, Wells, Ships, Hills, Clouds, Thunder and Lightning, Atmosphere, Sun, Moon, Mountains in the Moon, Shadows of Earth and Moon, Beginning and End of Eclipses, &c.

In which is also shewn,

A Curious *Trigonometrical Method* of discovering the Places where BEES hive in large Woods, in order to obtain, more readily, the *salutary Produce* of those little *Insects*.

By the Rev. Mr. TURNER, late of *Magdalen-Hall, Oxford*,
Author of *The View of the Earth*;—*View of the Heavens*;—*System of Gauging*;—
and *Chronologer Perpetual*.

*Cuncta Trigonus habet, satagitque docta Mathesis,
Ille aperit clausum, quicquid Olympus habet.*

*Within the grand Triangle lies unveil'd,
What Sages sought for, and what Heaven conceal'd.*

L O N D O N:

Printed for S. CROWDER, in Pater-noster-Row; and S. GAMIDGE,
Bookseller, in Worcester. MDCCLXV.

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A Concise & Experimental Method of discovering the Places where Bears live in large Woods, in order to obtain more readily, the necessary Proportions of those parts.

By the Rev. Mr. TURNER, late of St. John's Hall Oxford,
Author of The Ware of the Earth;—Theory of the Elements;—System of Gauging;—
and Christian's Repository.

Quarto. Twelve shillings. (Half-bound in leather.)
The above is the price of the book.

Within the grand Triangle the method
of finding the height of, and other things connected.

L O N D O N

Printed at S. Crowder, in Peter-nor-Row; and S. Cambridge,
Book-Seller, in Worcester. MDCCLXXV



TO THOSE
GENTLEMEN,

Whose *Genius* may incline, or *Employment* lead them to the
Study of the

MATHEMATICKS.

GENTLEMEN,

TRIGONOMETRY has always been look'd on as one of the most useful Branches of *Mathematical* Learning. *Navigation, Surveying, Astronomy, &c.* stand wholly upon this *Basis*. But the common Method of answering these Problems being by large Tables of *Sines, Tangents, and Secants*, renders it not only expensive by the purchase of them; but often precarious in the Solution, by the Mistakes of the Press. I have therefore, for the Use of the *Young Mathematician*, (from a Consideration of what has been published on this curious Subject) compos'd the present *System*, by which any of the *Cases* in *Right* or *Oblique* Plain Triangles may be answered on the Spot, by an easy Calculation in *Arithmetick* only. The great Advantages resulting from this Method to *Gentlemen* in the *Army* or *Navy*, as well as to those in their *private Studies* at *Home*, must immediately appear; as it will be found to answer the most necessary Problems as *expeditiously* as *Logarithms*; and at the same Time wholly deliver you from those *voluminous Tables* and the *inartificial Fatigues* of carrying them always with you.--- Should this little Treatise be so happy as to meet your Approbation, it will give a particular Pleasure to,

Your most humble Servant,

The AUTHOR.

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Study of the

MATHEMATICS.

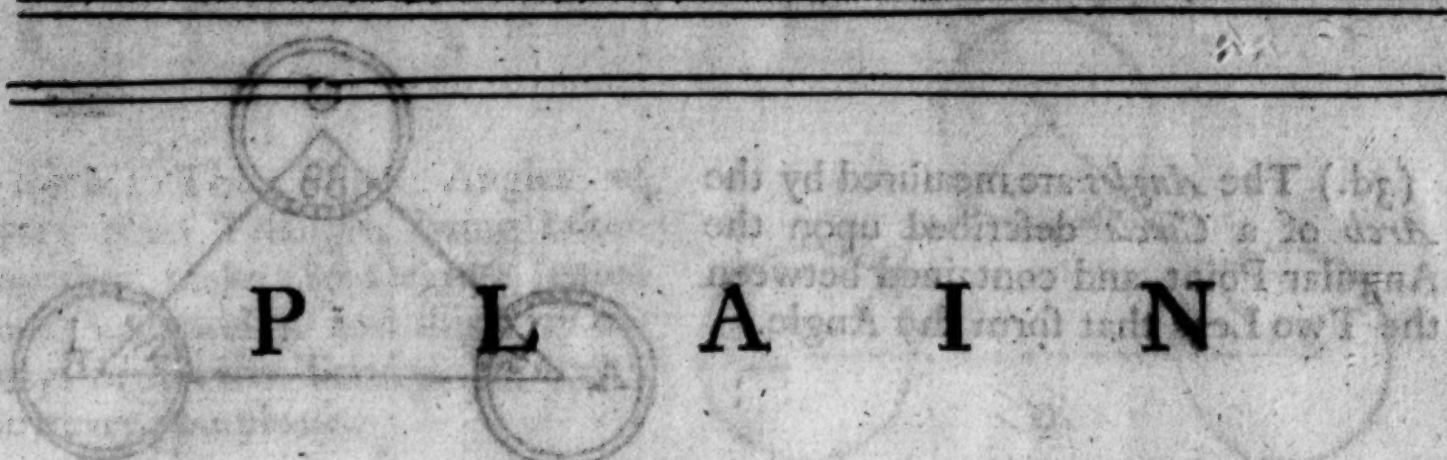
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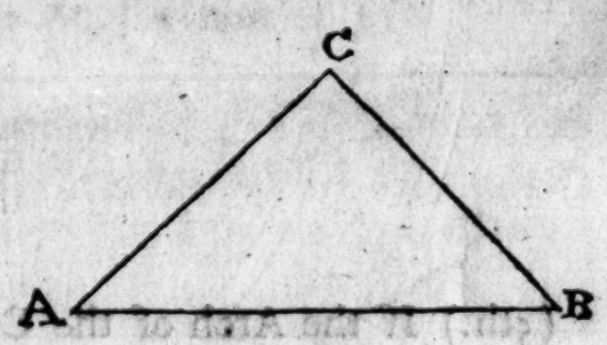




TRIGONOMETRY.

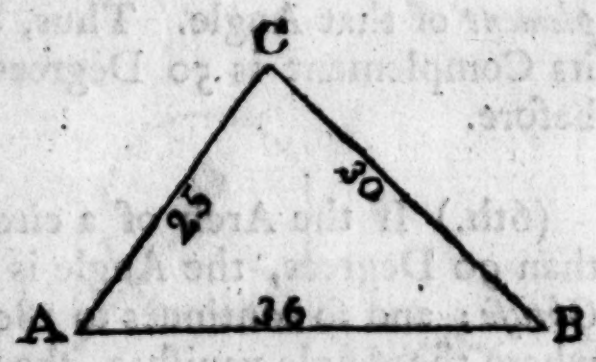
TRIGONOMETRY is that Part of *Mathematicks*, which is employed in calculating the *Sides* and finding the *Angles* of any *Triangle* required; it is of the greatest *Use* in *Life*, as nothing in *Navigation*, *Astronomy*, &c. can be done without it; and depends on the Knowledge of the following Observations.

(1st.) Every Triangle consists of Six Parts; that is, of *Three Sides* and *Three Angles*, as in the Figure ABC; the Three Sides are, AB, AC, CB, and the Three Angles, A, B, C.



NOTE. Sometimes an *Angle* is expressed by *Three Letters*; in that Case, the *Middle Letter* denotes the *Angular Point*. Thus, ABC expresses the Angle B; BAC the Angle A; and ACB the Angle C.

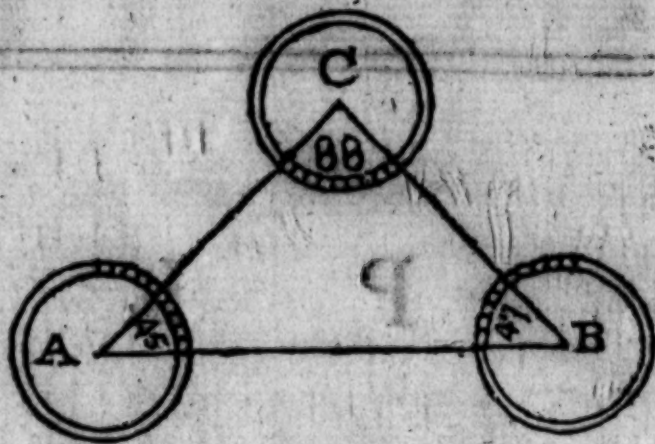
(2d.) The *Sides* of all plain Triangles are measured by a Line of equal Parts, as of *Inches*,—*Feet*,—*Yards*,—or *Leagues*.



Thus, The Side AB is 36 Leagues.—The Side AC 25 Leagues.—And the Side BC is 30 Leagues.

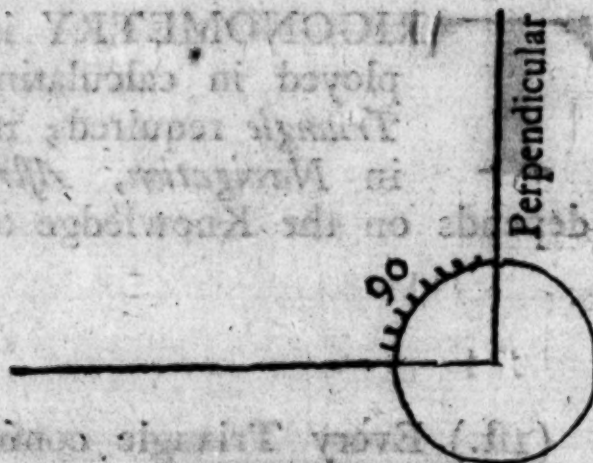
B (3d.) The

(3d.) The *Angles* are measured by the *Arch* of a *Circle* described upon the *Angular Point*, and contained between the *Two Legs* that form the *Angle*.

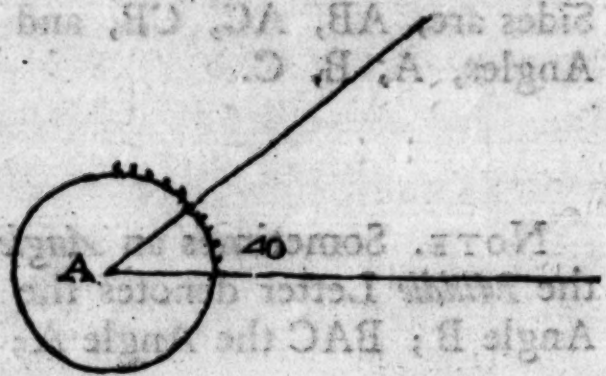


NOTE. Every *Circle* is divided into 360 equal *Parts*, called *Degrees*; each of which are divided into 60 more, called *Minutes*: And the *Number* of *Degrees* contained between the *Two Legs*, that constitute the *Angle*, is the *Measure* of that *Angle*. Thus, The *Angle A* is 45 *Degrees*.—The *Angle B*, 47.—The *Angle C*, 88.

(4th.) If the *Arch* of a *Circle* intercepted between the *Two Legs* be exactly 90 *Degrees*, the *Angle* is called a *Right Angle*, and the *Legs* are *perpendicular* to one another.

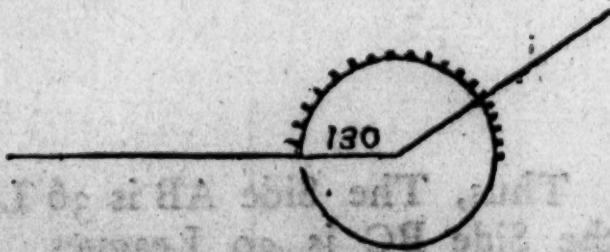


(5th.) If the *Arch* of the *Circle* be less than 90 *Degrees*, the *Angle* is said to be *Acute*.



NOTE. What an *Acute Angle* wants of 90 *Degrees*, is called the *Complement* of that *Angle*. Thus, Suppose the *Angle A* was 40 *Degrees*; then its *Complement* is 50 *Degrees*; for 40 added to 50 make 90, as observed before.

(6th.) If the *Arch* of a *circle* be more than 90 *Degrees*, the *Angle* is said to be *Obtuse*; and so continues to 180 *Degrees*, where the *Angle* vanishes, the *Lines* becoming *Strait*.

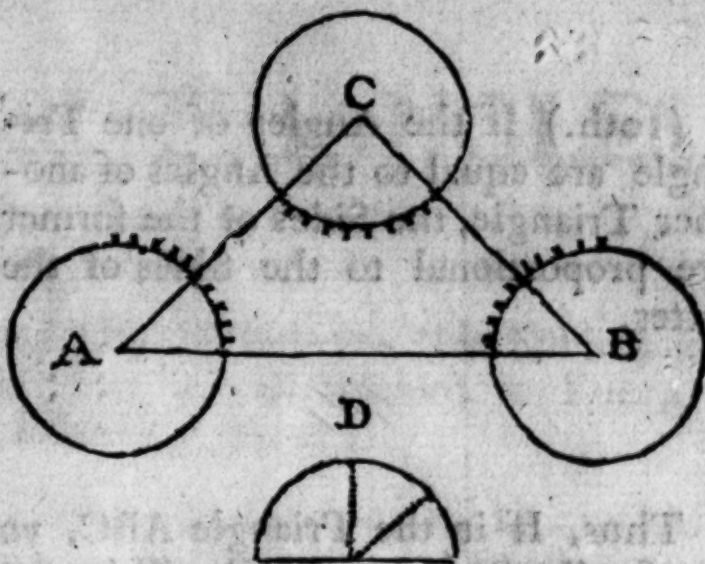


NOTE. What an *Obtuse Angle* wants of 180 *Degrees*, is also called the *Complement* of that *Angle* to a *Semicircle*.

(7th.) The



(7th.) The Three Angles of every plain Triangle, being taken together, make 180 Degrees (equal to a Semicircle), and this they always do, let the Triangle be drawn however you please.

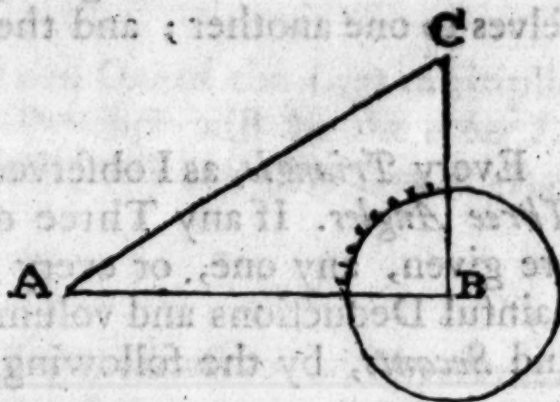


Thus, If the *Semicircle D* be drawn with the same *Radius*, or opening of the Dividers, as the little Circles on the Angles A, B, C, are; you will find, by taking off the several Arches, and applying them to the Semicircle, that they will just fill it up, and thereby make 180 Degrees; because every Semicircle contains that Number of Degrees.

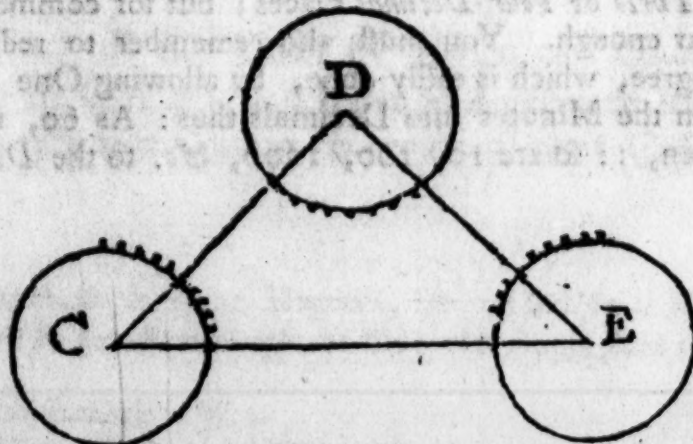
Hence it is evident, that if one Angle be a *Right One*, the other Two will be *Acute*; and taken together, be equal to one *Right Angle*, or just 90 Degrees.

Hence also, if *Two* Angles of any Triangle are known, the *Third* is easily found, being only the *Degrees* the other Two Angles want of 180.

(8th.) If a *Triangle* has one *Right Angle*, it is called a *Right Angled Triangle*: Thus ABC is a *Right Angle Triangle*, Right-angled at B.—In all *Right Angle Triangles*, the longest Leg is called the *Hypotenuse*;—the Leg on which it stands, the *Base*;—and the other Leg, the *Perpendicular*.

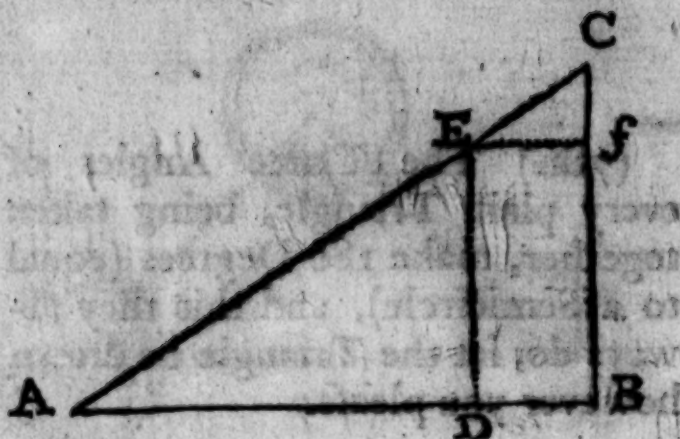


(9th.) If neither of the Angles is a *Right One*, then it is called an *Oblique Angled Triangle*; as the Triangle CDE is an *Oblique Triangle*.



(10th.) If

(10th.) If the Angles of one Triangle are equal to the Angles of another Triangle, the Sides of the former are proportional to the Sides of the latter.



Thus, If in the Triangle ABC, you draw the Line ED parallel to CB, the smaller Triangle ADE will be *similar to*, i. e. will have the same Angles with the larger Triangle ABC.—It will therefore always hold, as

$$AB : AD :: BC : DE.$$

$$\text{Or, as } AB : BC :: AD : DE.$$

$$\text{Or, as } AD : DE :: AB : AC.$$

$$\text{Or, as } AD : DE :: Ef : fC, \&c.$$

(11th.) In all Triangles the *greatest Side* is opposite to the *greatest Angle*; and on the contrary, the *greatest Angle* is opposite the *greatest Side*.—If Two Sides are *equal*, the opposite *Angles* are *equal*.—If all the Sides are equal, then all the Angles are equal to each other.

(12th.) In all Triangles, every Side is in proportion to its opposite Angle, and every Angle to its opposite Side: And further, as the Angle opposite to one Side, is to the Angle opposite the other Side, so are the Sides themselves to one another; and the contrary, the Sides to the Angles.

Every Triangle, as I observed before, consists of Six Parts—*Three Sides* and *Three Angles*. If any Three of the Six Parts (excepting the *Three Angles*) are given, any one, or every one of the rest may be found, without the painful Deductions and voluminous Tables of *Logarithms*, *Sines*, *Tangents*, and *Secants*, by the following *Rules* and *Axioms* *.

* This Method will be found as exact, as that by the *Logarithms*, if you carry on the Operation to *Three* or *Four Decimal Places*; but for common Purposes, *One* or *Two Decimal Places* will be near enough. You must also remember to reduce the *Minutes* of the Angles to *Decimals* of a Degree, which is easily done, by allowing *One tenth* for every *Six Minutes*.—Or you may turn the *Minutes* into *Decimals* thus: As 60, the *Minutes* in one Degree, : are to the *Minutes* given, :: so are 10, 100, 1000, &c. to the *Decimal* required.

Of Right Angled TRIANGLES.

THERE are generally reckoned by Writers on this Subject Seven Cases; but by this Method they are all reduced to Four; the Solutions of which depend on the following *Axioms*.

AXIOM I. Divide 4 Times the Square of the Complement of the Angle, whose opposite Side is either given or sought, by 300 added to 3 Times the said Complement; this Quotient added to the said Angle, will give you an *Artificial Number*, called sometimes the *Natural Radius* *, which will ever bear the same Proportion to the Hypothenufe, as that Angle bears to its opposite Side. — In Angles under 45 Degrees, the *Artificial Number* may be found easier thus: Divide 3 Times the Square of the Angle itself, whose opposite side is given or sought, by 1000; the Quotient added to 57.3 †, a fixed Number, that Sum will be the *Artificial Number* required. — This is to be used, when the Angles and a Side are given, to find another Side.

AXIOM II. The Square of both the Legs, i. e. the Square of the *Base* and *Perpendicular* added together, is equal to the Square of the *Hypothenufe*; whose Root is the *Hypothenufe* itself. — This is made use of, when the *Base* and *Perpendicular* are given, to find the *Hypothenufe*.

AXIOM III. The Sum of the *Hypothenufe* and One of the *Legs* multiplied by their *Difference*, the Square Root of that Product will be the other *Leg* required. — This comes into use, when the *Hypothenufe* and One *Leg* is given, to find the other *Leg*.

AXIOM IV. Half the *Longer* of the *Two Legs*, added to the *Hypothenufe*, is always in Proportion to 86 ‡, as the *Shorter Leg* is to its opposite Angle. — This is useful, when the Sides are given, to find the Angles.

NOTE. These 4 *Axioms* will answer all the Cases of *Right* and *Oblique Angled Triangles*, except the last Case in *Obliques*, which will require some further Assistance, and will be shown when we come to treat of that Case.

* The *Natural Radius* is only turning the *Right Angle*, = 90 Degrees, into an *artificial Number*, which shall always bear the same Proportion to the *Hypothenufe*, as the given Angle does to its opposite Leg.

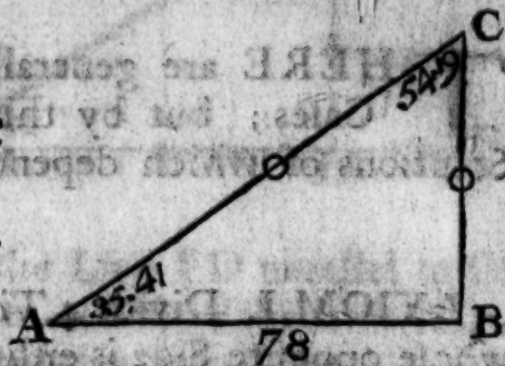
† 57.3 is the *Radius* of a Circle whose Circumference is 360.

‡ 86 = *Radius and Half* of a Circle whose Circumference is 360.

CASE I.

The Acute Angles, and one Leg given; to find the Hypothenufe and the other Leg.

Given $\left\{ \begin{array}{l} \text{Angle A} = 35.41 \\ \text{Angle C} = 54.19 \\ \text{Base AB} = 78 \end{array} \right\}$ to find $\left\{ \begin{array}{l} \text{Hypothenufe AC} \\ \text{and} \\ \text{Perpendic. BC.} \end{array} \right.$



(1st.) Find the Natural Radius
by Axiom I.

35.7	35.7
35.7	3
2499	107.1
1785	300.
1071	407.1

1274.49
4

407.1 5097.96 (12.5
4071 (54.3

10269 66.8 Natural Radius
8142

21276

20355

921

(2d.) Find the Hypothenufe
by Axiom I.

Angle C : Base :: Nat. Rad.
As 54.3 — 78 — 66.8

5344
4676

54.3 5210.4 (95.9 + Hypothenufe
4887

3234
2715

5190

4887

303

(3d.) Find the Perpendicular by Axiom III.

96 To Hypothenufe
78 Add the Base

174 Sum multiply
18 by Difference

1392
174

Extract the Root 3132 (55.9 + Perpendicular

25
105632
525

1109 10700

9981

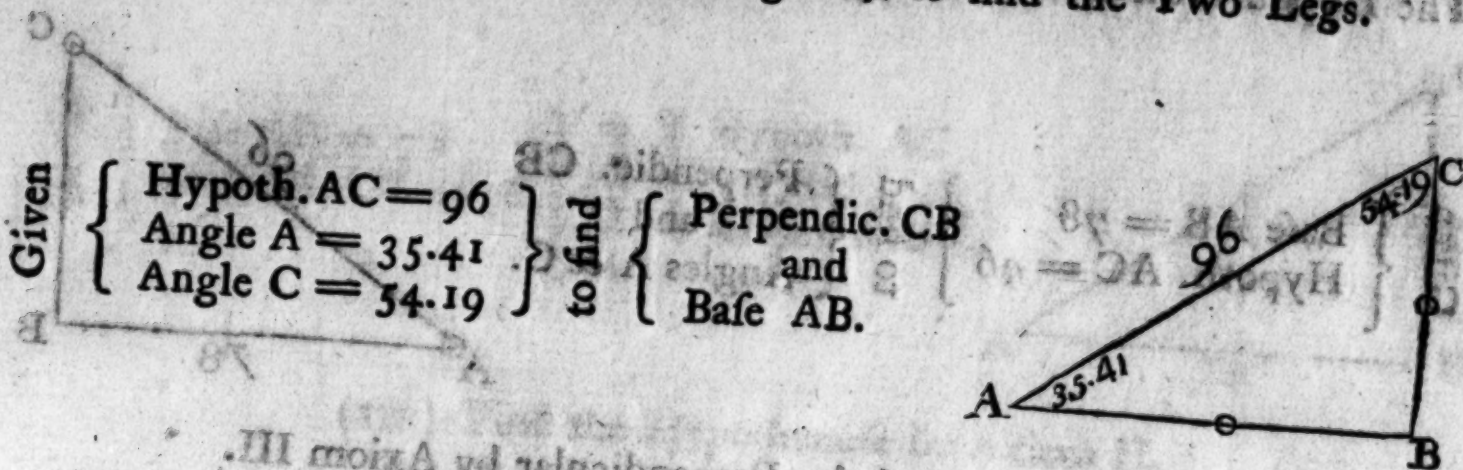
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Answer, $\left\{ \begin{array}{l} \text{Hypothenufe, } 95.9 +, \text{ or } 96. \\ \text{Perpendicular, } 55.9 +, \text{ or } 56. \end{array} \right.$

CASE

CASE II.

The Hypothenufe and Angles given, to find the Two Legs.



Given { Hypoth. AC = 96
Angle A = 35.41
Angle C = 54.19 } find { Perpendic. CB
and
Base AB.

(1st.) Find the Natural Radius by Axiom I.

35.7
35.7
—
2499
1785
1071
—
1274.49
3
—
1.000)3.823.47
57.3 Add.
61.1 Natural Radius

(2d.) Find the Perpendicular by Axiom I.

Nat. Rad. : Hypoth. :: Angle A
As 61.1 = 96 = 35.7
= 96 = 96
2142
3213
61.1)3427.2(56 Perpen
3055
—
3722
3666
—
56

(3d.) Find the Base by Axiom III.

96 To Hypothenufe
56 Add Perpendicular
—
152 Sum multiplied by
40 The Difference

Extract the Root 6080(77.9 + Base

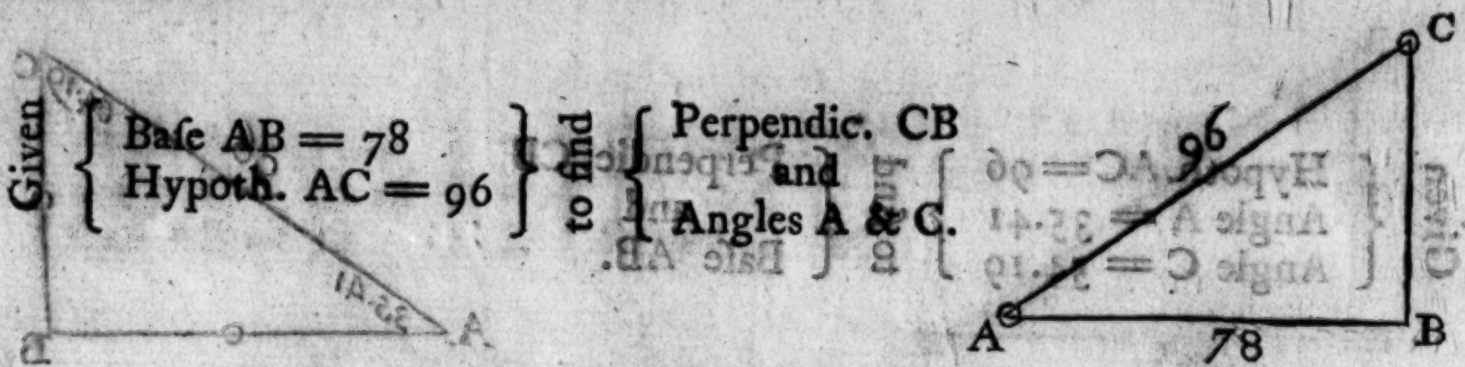
49
—
147)1180
1029
—
1547)15100
13941
—
1169

Answer, { Perpendicular, 56.
Base, 77.9 +, or 78.

PLAIN TRIGONOMETRY

CASE III.

The Hypotenuse and One Leg given, to find the Angles and the other Leg.



(1st.) Find the Perpendicular by Axiom III.

96 To Hypotenuse
78 Add the Base

174 Sum multiply
18 By Difference

1392

Extract the Root 3132 (55.9 + Perpendicular)

25

105632

525

110910700

9981

719

(2d.) Find the Angle by Axiom IV.

To Hypotenuse 96
Add half longer Leg 39

Fixed Number :: Perpendicular

Sum 135

56

86

336

448

1354816. (35.57 + Angle A)

405

766

675

910

810

1000

945

55

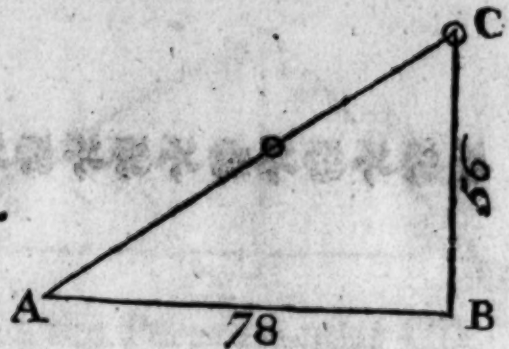
Answer, $\left\{ \begin{array}{l} \text{The Angle, } 35^\circ 41' \text{ nearly.} \\ \text{The Perpendicular, } 55.9 +, \text{ or } 56. \end{array} \right\}$

CASE

CASE IV.

The Two Legs given; to find the Hypothenufe and the Angles.

Given { Base AB = 78
Perpend. CB = 56 } Find { Hypoth. AC
and
Angles A & C.



(1st.) Find the Hypothenufe by Axiom II.

Perpendicular 56 78 Base
squar'd 56 78 squar'd

336 624
280 546
3136 9084 Square of Base
3136 Square of Perpendic.

Extract the Root 9220(96 The Hypothenufe

81

136(1120
1116

To Hypothenufe 96
Add half longer Leg 39

Sum 135

Fixed Number 86

Perpendicular

56
86

336
448

135)4816.(35.674+ Angle A

405

766
675

910
810

1000
945

550
540

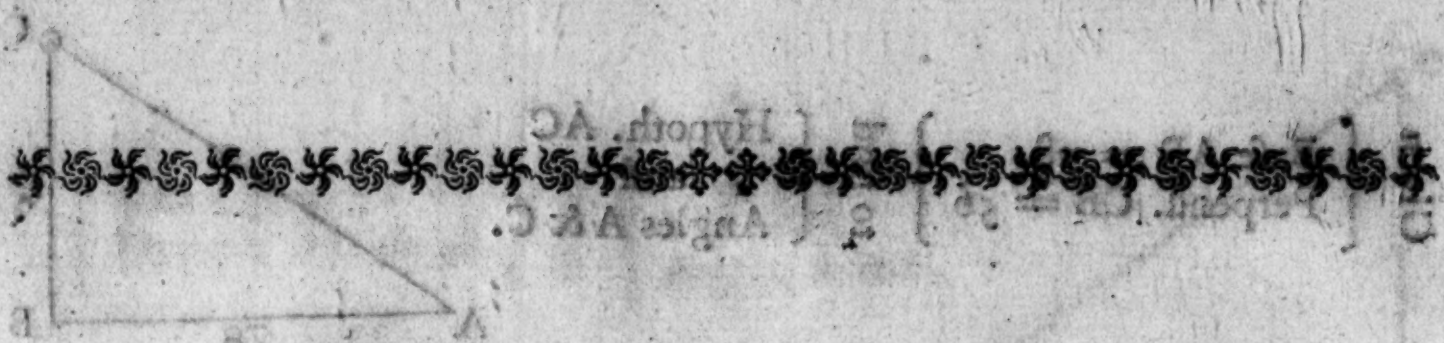
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Answer, { The Hypothenufe, 96.
The Angle, 35°.674, 35° 41' nearly.

D

NOTE.

NOTE. Thus all the Cases of *Right Angled Triangles*, are easily and readily answered: and, by the same Rules, and with the like Ease may the *Oblique Angled Triangles* be answered, as will evidently appear in the following Cases.



Of Oblique TRIANGLES.

IN the Solution of an *Oblique Triangle*, it is necessary, by this Method, to divide it into Two *Right Angled Triangles*, by means of a *Perpendicular*, which must always fall upon the End of a given Side, and opposite to a given Angle.

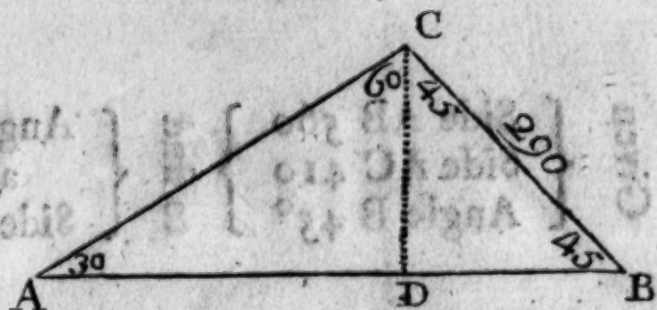
By this means the *Perpendicular* will sometimes fall *within*, and sometimes *without* the Triangle: When it falls within, it falls upon some Part of the Base, or longest Side; but when it falls without, it falls upon one of the shorter Sides continued. In either Case, there are Two *Right Angled Triangles* made, and the Angles, or Sides sought, are found as if they were Parts of a *Right Angle*, by the foregoing Axioms; but it requires Two or Three Operations.

CASE

CASE I.

Two Angles, and a Side opposite to one of them, given; to find the other two Sides.

Given { Angle A 30
Angle B 45
Side BC 290 } to find { Side AC
and
Side AB.



(1st.) Find the Perpendicular CD.

N. Rad.	Hypoth.	∠ at B
As 63.6	290	45
	45	
	1450	
	1160	
	63.6)13050(205	Perpendicular CD
	1272	
	3300	
	3180	
	120	

(2d.) Find the Side AC.

∠ at A	Perp. CD	N. Rad.
As 30	205	60
	60	
	30)12300(410	The Side AC Hypothenufe
	120	
	30	
	30	
	0	

(4th.) Find the Side DB.

To Hypoth. CB	290
Add Perp. CD	205
Sum	495
Multiply by Difference	85
	2475
	3960

Extract the Root 42075(205 DB

4
405)2075
2025
50

45	45
45	3
225	135
180	300
2025	435
4	
435)8100(18.6	
434	45
3750	63.6 Natural Radius
3480	
2700	30
2610	30
	900
	3
	2.700
	57.3
	Natural Radius 60.0

(3d.) Find the Side AD.

To Hypoth. AC	410
Add Perp. CD	205
Sum	615
Multiply by Difference	205
	3075
	12300
Extract the Root	126075(354.9 AD
	9
	65)360
	325
	704)3575
	2819
	7089)75900
	63801
	12099

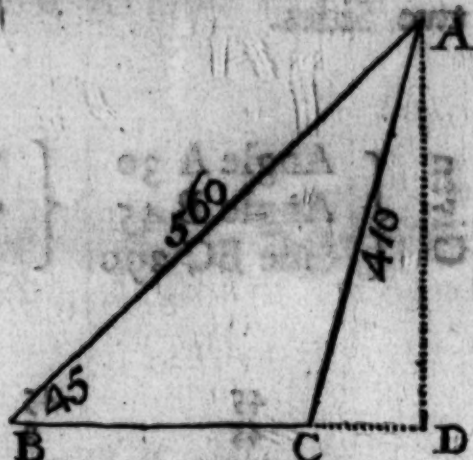
Answer, { The Side AC 410.
The Side AB 560.
The Angle C 105.

CASE

CASE II.

Two Sides, and an Angle opposite to One of them, being given; to find the rest.

Given { Side AB 560
Side AC 410
Angle B 45° } to find { Angle C
and
Side BC.



(3d.) Find the Angle DAC.

To Hyp. CA 410
Add $\frac{1}{2}$ Perp. AD 198

As 608 — Fixed Num. Side CD

As 608 — 86 — 106

86

636

848

608)9116(14.9+, or $15^\circ \angle A$

608

3036

2432

6040

3472

568

(1st.) Find the Side AD.

Nat. Rad. Hyp. BA 560 $\angle B$ 45

As 63.6 —

45

2800

2240

93.6)25200.0(396 AD

1908

6120

5724

3960

3816

144

(4th.) Find Side BD.

To Hypoth. BA 560
Add Perpend. DA 396

Sum 956

Multiply by Differ. 164

3824

5736

956

Extract the Root 156784(395.9+ or 396 BD

9..

69)667

621

785)4685

3925

7909)75600

71181

4719

Then from BD = 396

Take — CD = 106

Remains — BC = 290 Required.

Hence we find by Inspection,
The Angle ACD = 75°
The Angle ACB = 105°
and The Angle BAC = 30° Required.

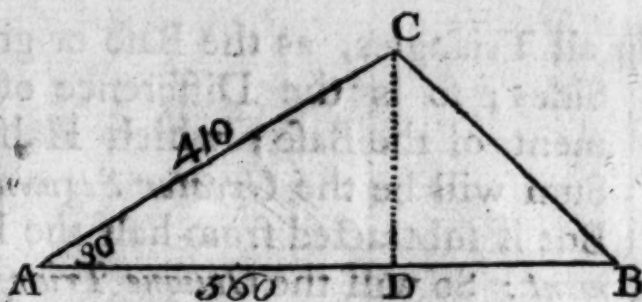
* In this, and several of the following Cases and Problems, the Operation for finding the Natural Radius is omitted, for Want of Room.

CASE

CASE III.

Two Sides, with the Angle comprehended by them, given; to find the rest.

Given { Side AC 410 } { Side BC }
 { Side AB 560 } { and }
 { Angle A 30° } { Angles B & C. }



(1st.) Find the Perpendicular CD.

N. Rad. Hypoth. AC 410 $\angle$ at A 30
 As 60 ——— 410 ——— 30
 60) 12300 (205 Perpendicular CD
 120
 300
 300
 0

(2d.) Find the Part AD.

To Hypoth. AC 410
 Add Perpend. CD 205
 Sum 615
 Multiply by Differ. 205
 8075
 12300
 Extract the Root 126075 (355 Base AD
 9...
 65) 360
 325
 705) 3575
 3525
 50
 From AB = 560
 Take AD = 355
 Remains BD = 205

(3d.) Find the Side BC.

BD squar'd 205 205 CD squar'd 205
 205
 1025 1025
 4100 4100
 To Square of DB 42025
 Add Square of CD 42025
 Extract the Root 84050 (289.9 +, or 290 BC
 4
 48) 440
 384
 569) 5650
 5121
 5789) 52900
 52101
 799

(4th.) Find the Angle B.

To 290 the Side BC
 Add 102 5 half DB
 Fix'd Num. Side DC
 As 392.5 ——— 86 ——— 205
 86
 1230
 1640
 3925) 17630.0 (44.9 +, or 45° $\angle$ B
 15700
 19300
 15700
 36000
 35325
 675

The Angle A = 30, added to Angle B = 45, and then subtracted from 180, leaves 105 for Angle C.

Answer, { The Side BC 290
 { The Angle B 45°
 { The Angle C 105°.

E

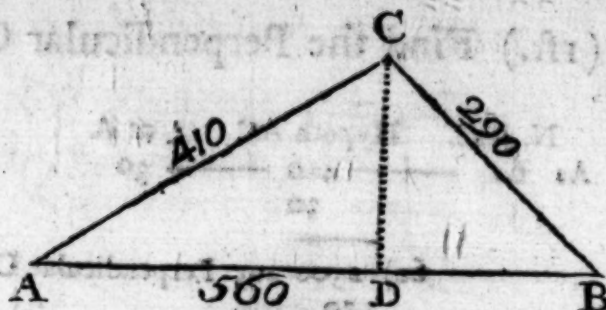
CASE

CASE IV.

The Three Sides given; to find the three Angles.

In all Triangles, as the Base or greater Side is to the Sum of the other Two Sides; so is the Difference of the Sides to the Difference of the Segments of the Base; which Half Difference, added to Half the Base, the Sum will be the *Greater Segment*, upon which the Perpendicular falls: But if subtracted from half the Base, the Remainder will be the *Less Segment*: So will the *Oblique Triangle* be reduced to two Right Angled ones, and may be answered after the same Manner as before.

Given { Side AB 560 { to find { Angle A
 { Side AC 410 { { Angle B
 { Side BC 290 { { Angle C.



(1st.) Find the Segments of the Base.

As 560 ——— 700 ——— 120
 120
 14000
 770
560)84000(150 Difference
 560
 2800
 2800
 0
To $\frac{1}{2}$ Base 280
Add $\frac{1}{2}$ Diff. 75
Gr. Seg. AD 355
From $\frac{1}{2}$ Base 280
Subtract $\frac{1}{2}$ Diff. 75
Less Seg. DB 205

(3d.) Find the Angle at A.

To 410 Side AC
Add 177 half AD
As 587 ——— 86 ——— 205
 86
 1230
 1640
387)17630(30 $\angle A$
 1761
 20

(2d.) Find the Perpendicular CD.

To Hypoth. CA 410
Add Gr. Seg. AD 355
Sum 765
Multiply by Difference 55
 3825
 3825
Extract the Root 42075(205 CD
 4
 405)2075
 2025
 50

(4th.) Find the Angle at B.

To 290 Side BC
Add 102 half DB
As 392 ——— 86 ——— 205
 86
 1230
 1640
392)17630(44.9 +, or 45° $\angle B$
 1568
 1950
 1568
 3820
 3528
 292

Then the Angle A = 30°, added to B = 45°, and subtracted from 180°, leaves 105° for the Angle C, which were the Angles required.

The

The Use of TRIGONOMETRY.

Exhibited in the Solutions of a Number of interesting Problems; many of which every Day occur; are of the greatest Utility in the Army, Navy, &c. and cannot be answer'd without it.

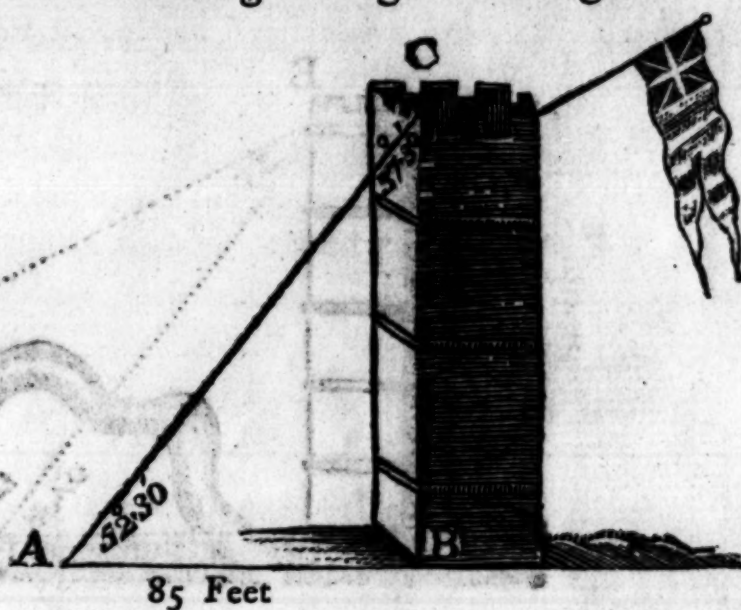
PROBLEM I.

To take the Height of any *accessible Object* at one Station.

First, with a *Quadrant*, by looking through the Sights to the Top of the Tower, find the Quantity of the Angle A, which suppose $52^{\circ} 30'$; then measure the Distance AB, which suppose to be 85 Feet; from hence you may proceed to find the *Height*, by Case I. of Right Angled Triangles.

37.5
37.5
—
1875
2625
1125
—
1406.25
3
—
1000)4.218.75
57.3
—
61.51875 Natural Radius

But 61.52 is exact enough in Practice.



(2d.) Find the Perpendicular BC.

To Hypothense 139.44.
Add Base 85.

Sum 224.44
Multiply by Difference 54.44
89776
89776
89776
112220

(1st.) Find the Hypothense AC.

Angle C : Base :: Nat. Rad.
As 37.5 — 85 — 61.52

85
30760
49216

37.5)5229.20.(139.44 + Hyp.

375
1479
1125
—
3542
3375
—
1670
1500
—
1700
1500
—
200

Extract the Root 12218.5136(110.537 Perpendic.

1.....
21)22
21
—
2205)11851
11025
—
22103)82636
66309
—
221067)1632700
1547469
—
85231

Answer, 110.537 +, or 110 Feet, and above $\frac{1}{2}$: The Height Required.

NOTE; That in this, and all such Cases, you must add the *Height* of your *Eye*, or *Instrument* to the *Altitude* before found.

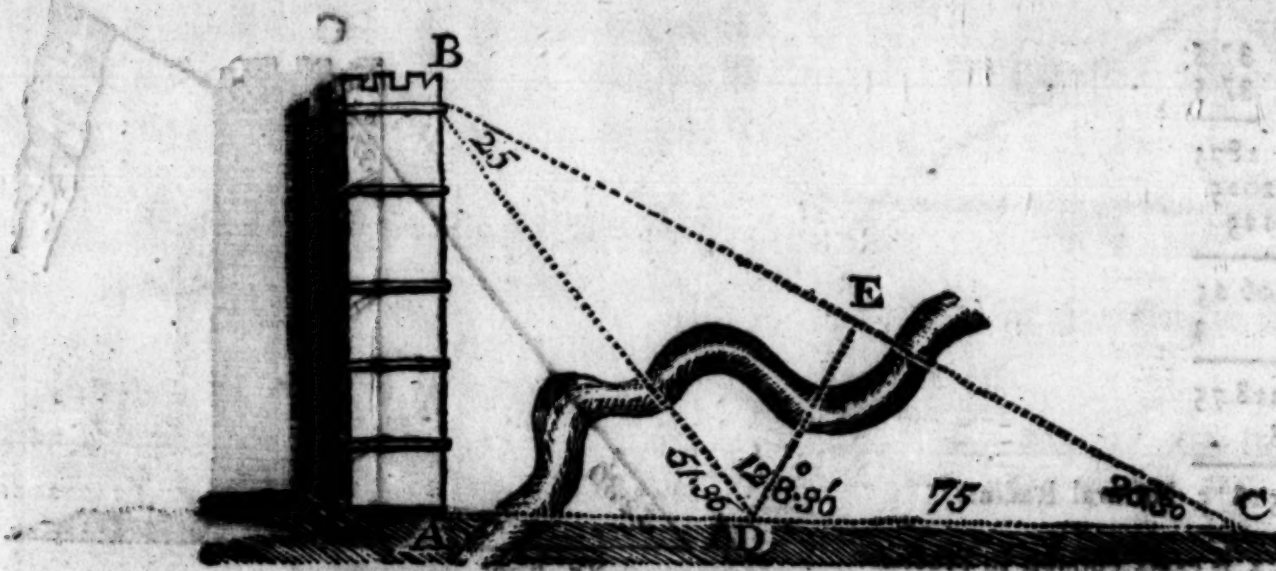
PROBLEM

PROBLEM II.

To measure an *Inaccessible Altitude*.

Let AB, in the following Figure, be a *Church, Tower, or Fort*, whose Height is required; but by Reason of a River, or some other *Obstacle*, it is *inaccessible*; that is, you cannot come to the Foot of it, by Reason of the *Water, &c.*

First, with a *Quadrant*, take the Angle of *Altitude* at C, which suppose $26^{\circ} 30'$. Then measure in a Right Line towards the *Tower* to D, any Distance, suppose 75 Feet, and at D observe again the Angle of *Altitude*, which let be $51^{\circ} 30'$.



Then; the two Visual Lines CB and DB, with the Distance DC, make the Oblique Triangle CBD, in which are given—All the Angles and Side CD. The Angles BCD being $26^{\circ} 30'$, and the Complement of ADB $51^{\circ} 30'$ to 180, is the Obtuse Angle BDC $128^{\circ} 30'$. Consequently, the third Angle CBD, at the Top, is $= 25^{\circ}$.

(1st.) Find the Perpendicular DE in Triangle DBC. (3d.) Find the Height AB in the Right Angled Triangle ABD.

Nat. Rad. : Op. Side DC :: Ang. C : Perp. DE
As 59.4 — 75 — 26.5 — 33.46

Nat. Rad. : Op. Side BD :: Ang. D : Height
As 65.7 — 79.19 — 51.5 — 62 AB

(2d.) Find the Visual Line BD in Triangle BDE. (4th.) Find the Distance AD in the Triangle ABD.

Ang. B : Op. Side DE :: N. Rad. : Side BD
As 25 — 33.46 — 59.17 — 79.19

N. Rad. : Op. Side BD :: Ang. ABD : Dif. AD
As 61.7 — 79.19 — 38.5 — 49.41

Answer, { 62 Feet the Height.
 { 49.41, or $49\frac{1}{2}$ Feet the Distance from the second Station.

NOTE. The Line BD is the Length of a Scaling Ladder, which would reach from the Station at D over the Foss or Ditch, to the Top of the Tower at B.

PROBLEM

PROBLEM III.

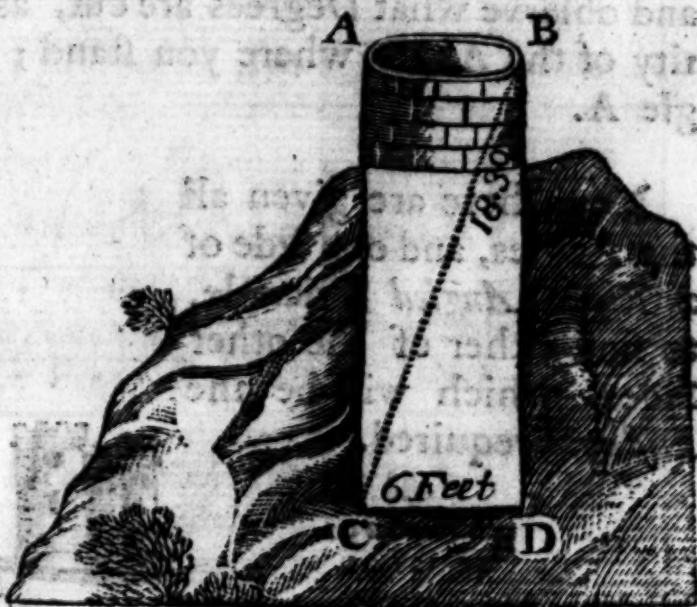
To measure the *Depth* of a *Well*, or the *Height* of an *Object* from the *Top* of it.

First, look through the Sights of the *Quadrant* to the Bottom of the opposite Side the Well at C, so you will have the *Angle* CBD; next, take the *Breadth* AB at the Top, which is equal to CD at the Bottom; Then, by Case I. of Right Angle Triangles, you may easily find the *Depth* BD required.

Suppose the Angle at B, by Observation, to be $18^{\circ} 30'$, and the Breadth at the Top 6 Feet; what's the *Depth*.

18.5
18.9
—
925
1480
185
—
342.25
3
—
1.026.75
57.3

Natural Rad. 58.32675 but 58.3 is enough.



(1st.) Find the Hypothenufe BC.

∠B Op. Side N. Rad.
As 18.5 — 6 — 58.3
18.5)349.8.(18.9 + the Line BC
185
—
1648
1480
—
1680
1665
—
15

Answer, { 17.89 +, or 18 Feet, the Depth required.

(2d.) Find the Depth BD.

To Hypoth. BC 18.9
Add Side CD 6.
—
Sum 24.9
Multiply by Differ. 12.9
—
2241
498
—
249
Extract the Root 321.21(17.89 the Depth AB
—
27)221
189
—
348)3221
2784
—
3569)43700
32121
—
11579

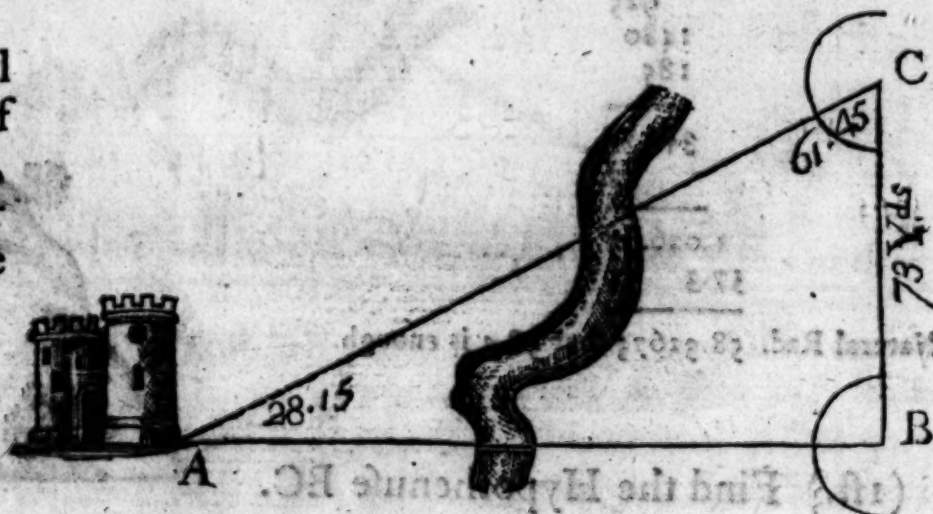
PROBLEM IV

To measure the *Distance* of any *Object*.

Suppose yourself standing at B, and a great Way off, as at A, you see a *Fort* or *Castle*, &c. or any other *Object*, whose *Distance* you would find from the Place where you now stand.

First, a *Theodolite*, or *Semicircle*, being placed at B, lay the *Index*, with its *Sights*, on the *Diameter*, where the Degrees begin, and through them view the *Castle*, &c. at A. The Instrument remaining fix'd in this Position, move the *Index* to 90 Degrees, (that being a Right Angle) and view some Mark at a Distance, (the farther off the better) as at C.—Next measuring the Distance from B to C, which suppose 73 Yards, remove your Instrument, and set it up at C. Then, with the *Index* laid upon the Beginning of the Degrees, as before, turn the Instrument about, till you can see your *first Station* at B, where fasten it; then turn the *Index* till you can see the *Object* A, and observe what Degrees are cut, as suppose $61^{\circ} 45'$, which is the Quantity of the Angle where you stand; whose Complement to 90° is the Angle A.

Now, here are given all the Angles, and one Side of a *Right Angled Triangle*, to find either of the other Sides, which will be the *Distance* required.



28.25
28.25

14125

5650

22600

5650

798.0625

3

2.394.1875

57.3

59.69 or 59.7 Natural Radius

(2d.) Find the Distance from B.

To Hypoth. AC 154.2

Add Side BC 73

Sum 227.2

Multiply by Differ. 81.2

4944

2272

19176

(1st.) Find the Distance from C.

Extract the Root 184.8.64 (135.8 +, Dist. from B)

Ang. A : Op. Side BC :: N. Rad. : Dist. CA
As 28.25 — 73 — 59.7 — 154.2

23)84

69

265)1548

1325

2709)21664

21664

700

yards
Ans. { 135.8 +, or 136 Dist. from B.
 { 154.2, or 154 Dist. from C.

PROBLEM

PROBLEM V.

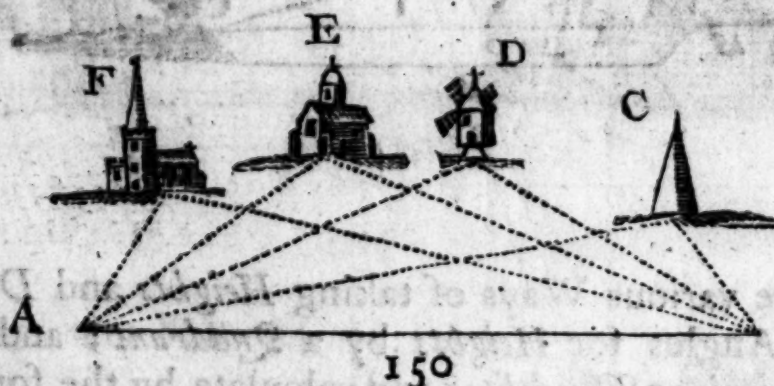
To take the *Distances* of several *inaccessible Objects*, as *Forts, Churches*, in a *Town*, or *Squadron of Ships at Sea*, and to delineate them upon *Paper*.

First, make choice of two Places, from either of which you may conveniently see all the *Objects*; which two Places let be A and B in the following Figure.—This being done, set up your Instrument at A, laying the *Index* on the *Diameter*, and turn the whole Instrument about, till, through the Sights, you see your second Station B. Then, fixing the Instrument, direct your Sights to the several *Objects*, C, D, E, and F; noting down the Degrees cut at each Observation, which suppose to be as in the Table.

Then, remove the Instrument to B, laying the *Index* on the *Diameter*, and turn it about, till, through the Sights, you see your former Station at A; then direct your Sights to every one of the *Objects* at C, D, &c. setting down the Degrees at each Observation, as in the Table. Also measure the *Stationary Distance*, and set that down.

	C	D	E	F
1st Station	11	23	36	59
2d Station	51	31	22	13
Stationary Distance 150 Yards				

First, upon a Piece of Paper draw the Line AB; and from a *Scale of equal Parts*, take off, with your Dividers, the *Stationary Distance* = 150, and set it from A to B, so will A represent your *first Station*, and B the *second*. Then lay the *Center* of the *Protractor* upon the Point A, with its *Diameter* upon



the Line AB; keeping it fast, make Marks by the Edge at 11, 23, 36, 59, and draw Lines from the Point A through each of those Marks. Then upon B place the *Center* of your *Protractor*, its *Diameter* lying upon the Line AB; make Marks by the Side at 13, 22, 31, 51. Then draw Lines from the Point B through each of these Marks, and where the Lines cut the former *correspondent Lines* there will be found the Places representing these *Objects*. Then any of these Lines being taken in a Pair of Dividers, and applied to the Scale you laid your *Stationary Distance* down by, will give you their *Distances*, either from your *Stations* or from one another.

The *Distance* of any of these *Objects* from either Station, &c. may be found by *Calculation*; one Side and the *Angles* being given: But I shall omit that, on purpose to exercise the *Learner's Genius*, and proceed.

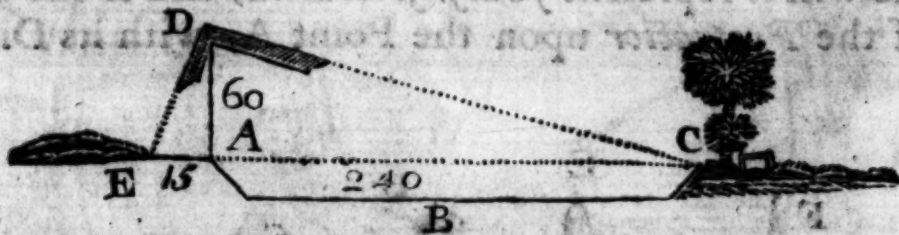
PROBLEM

PROBLEM VI.

To take the *Distance*, upon level Ground, of any *inaccessible Tree, Fort, &c.* or *Breadth of a River*, by a *Common Square*.

Suppose there is a River, as A B C, whose *Breadth* you want to know. —First, upon the Bank, at A, set up a Stick, AD, which suppose to be 5 Feet, or 60 Inches high; then fixing your *Square* on the Top, at D, look by the Side of it till you see the Edge of the opposite Shore C, and fasten it, as it may not go from that Position. This done, extend a Thread from D, by the other Side of the *Square* till it touch the Ground at E. Then measure the Distance EA, which suppose 15 Inches, (or 1 Foot 3 Inches) and you may find AC (by Reason of similar Triangles) thus.

Dist. EA : Side DA :: Side DA : Dist. AC
As 15 ——— 60 ——— 60 ——— 240 Inches,
which, reduc'd to Feet, give 20 for the Breadth of the River sought.



NOTE. There are various Ways of taking *Heights* and *Distances*; but the best is to take the Angles for *Heights* by a *Quadrant*; and the Angles for *Distances* by a *Semicircle* or *Theodolite*; and calculate by the foregoing *Axioms*. In all *Heights* the *Triangle* stands upright; but in *Distances*, it is suppos'd to lie flat or horizontal.

I shall here shew the Learner, how to take the *Breadth of a River*, or a *small Distance*, without any *Instrument whatever*; which is thus. Standing upon the Bank, bring down the Edge of your *Hat*, till it appears to touch the opposite Side, then steady your Head by laying your Hand under your Chin, and turn yourself towards some level Ground, observing where the Edge of your *Hat* glances upon it; for then, the *Distance* from you to that Place, is equal to the *Breadth of the River*, or *Distance* required.

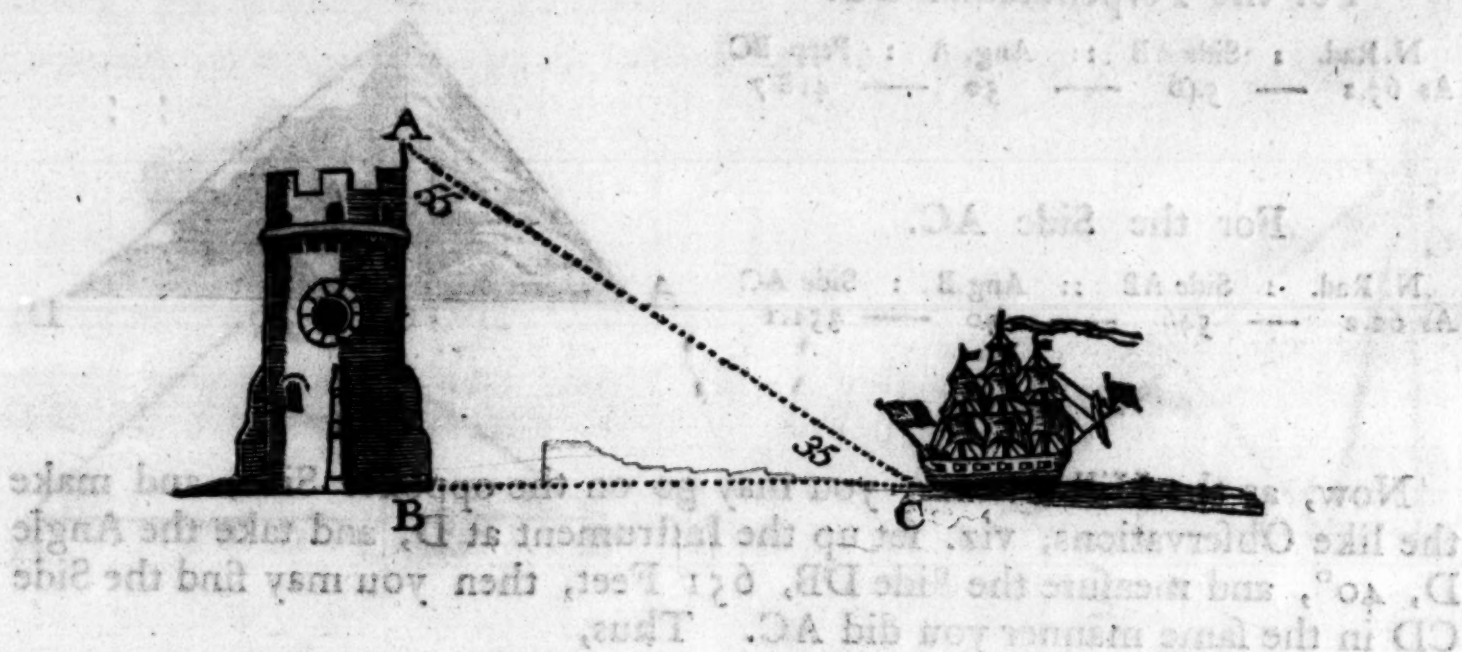
PROBLEM

PROBLEM VII.

To find, from the Top of a *Fort*, or *Tower*, how far any *Tree*, *Ship*, &c. is from you.

Let *A* be the Top of a *Tower* or *Castle* standing by the *Sea-side*; and let *C* be a *Ship* at Sea, or lying at Anchor, and you would know how far that *Ship* is off the *Castle-wall*.

With your *Quadrant* or *Semicircle*, direct your Sights from the Top of the *Tower* to the Place where the *Ship* is, and take the Angle, which we will suppose to be 55 Degrees. Then the *Castle-wall* being known before to be 143 Feet high, you may easily find the *Distance* of the *Ship* from the *Wall* in this Manner.



(1st.) Find the Side *AC*.

$$\begin{array}{rclcl} \text{Ang. C : Height Wall :: Nat. Rad. : AC} \\ \text{As } 35 & \text{---} & 143 & \text{---} & 61 & \text{---} & 249.2 \end{array}$$

(2d.) Find the Base *BC*.

$$\begin{array}{rclcl} \text{N. Rad. : AC :: Ang. A : Dist BC} \\ \text{As } 67 & \text{---} & 249.2 & \text{---} & 55 & \text{---} & 204.56 \end{array}$$

By this Method you may easily discover if a *Fleet* of *Ships*, or one *single Ship*, at Sea, makes towards you or not. For having observ'd from the Top of the *Fort* the Angle from thence to the *Ship*, and noted it down, rest a little Time, and observe again: Then, if the Angle be *bigger* than before, the *Ship* is *departing* from you; but if *less* she is *making* towards you.

PROBLEM VII.

To take the *Perpendicular Height* of a *Hill* or *Mountain*, and also the *Horizontal Line*, or *Base*, on which it stands.

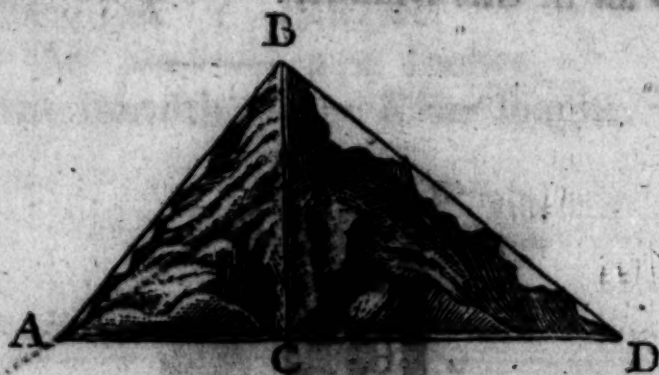
Let ABD be the *Hill*.—First, set up a Mark on the Top at B, equal to the Height of the *Quadrant* or Instrument that is used at the Bottom, from whence you intend to make your Observation. Then by looking through the Sights to B take the Quantity of the Angle at A, which we will suppose to be 50° . Next measure the Hill from A to B, which let be 546 Feet. This being done, you may easily find the *Perpendicular BC*, or Part of the *Base AC*, by Case II. of Right Angles.

For the Perpendicular BC.

$$\begin{array}{l} \text{N. Rad. : Side AB :: Ang. A : Perp. BC} \\ \text{As } 65.2 \text{ — } 546 \text{ — } 50 \text{ — } 418.7 \end{array}$$

For the Side AC.

$$\begin{array}{l} \text{N. Rad. : Side AB :: Ang. B : Side AC} \\ \text{As } 62.2 \text{ — } 546 \text{ — } 40 \text{ — } 351.1 \end{array}$$



Now, as the *Hill descends*, you may go on the opposite Side, and make the like Observations, viz. set up the Instrument at D, and take the Angle D, 40° , and measure the Side DB, 651 Feet, then you may find the Side CD in the same manner you did AC. Thus,

$$\begin{array}{l} \text{Nat. Rad. : Side DB :: Ang. B : Side CD} \\ \text{As } 65.23 \text{ — } 651 \text{ — } 50 \text{ — } 499 \end{array}$$

If to the Part AC = 351.1 Feet, be added the Part CD = 499 Feet, the Sum 850.1 Feet will be the whole Length of the *Horizontal Line AD* requir'd.

The *Perpendicular Height DC* is = 418.7 as above.

PROBLEM

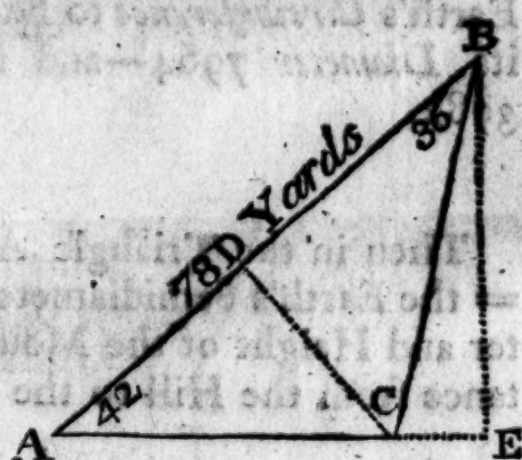
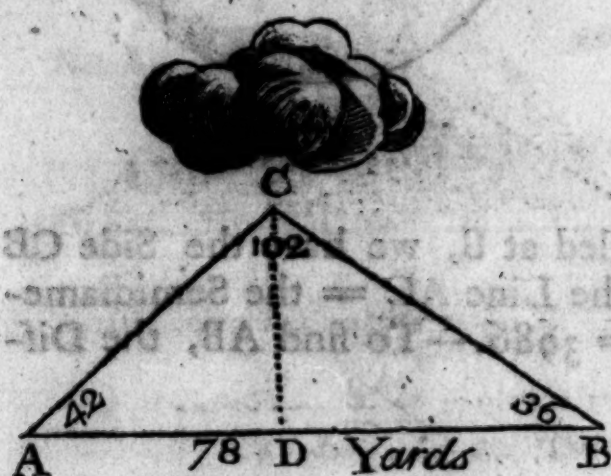
PROBLEM IX.

To take the *Height* and *Distance* of a *Cloud*.

Suppose it was requir'd to find the Height of the *Cloud* C.

Let a Person standing at A, look through the *Quadrant* to the *Cloud* at C, so will the Thread cut the Angle at A. At the same Time let another Person, making the like Observation at B, take the Angle B. Then measure the *Distance* between the *two Stations*. By this means you will have *one Side* and *all the Angles* of an *Oblique Angle Triangle* given, from whence you may easily obtain the rest, and particularly the *Perpendicular* CD, which will be the *Height* of the *Cloud* requir'd.

EXAMPLE. Suppose the Angle at A, by Observation, be 42° , the Angle B 36° , and the Distance AB 78 Yards: I demand the *Height* of the *Cloud*.



(1st.) Find the Perpendicular BE in Triangle ABE *.

$$\begin{array}{l} \text{N. Rad. : Op. Side AB :: Ang. A : Perp. BE} \\ \text{As } 62.6 \text{ --- } 78 \text{ --- } 42 \text{ --- } 52.1 \end{array}$$

(2d.) Find the Hypotenuse CB in Triangle CBE.

$$\begin{array}{l} \text{Ang. BCE : Op. Side BE :: N. Rad. : Hyp. BC} \\ \text{As } 78 \text{ --- } 52.1 \text{ --- } 79 \text{ --- } 53.2 \end{array}$$

(3d.) Find the Perpendicular CD in Triangle CDB:

$$\begin{array}{l} \text{N. Rad. : Op. Side BC :: Ang. DBA : Perp. CD} \\ \text{As } 61.1 \text{ --- } 53.2 \text{ --- } 36 \text{ --- } 31.3 \end{array}$$

Answer, 31.3 Yards, the Height requir'd.

* The Figure on the *Right Hand* is only that on the *Left* set in a different Position, to shew in a more natural or easy Manner, how the Perpendicular falls from the End of the given Side AB, upon the Side AC produc'd to E.

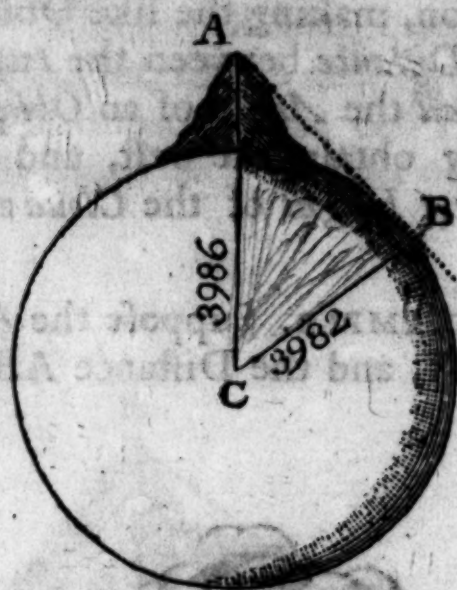
PROBLEM

PROBLEM X.

To find how far a *Hill* of any given *Height* can be seen at *Sea*, or upon *level Ground*.

How far, for Instance, can the *Pike of Teneriff* be seen at *Sea*, whose *Height* is about *four Miles*.

✧ The *Circumference* of the *Earth* is sup-
pos'd by *Mathematicians* to be divided into
360 equal *Parts*, called *Degrees*; and our
countryman, Mr. *Norwood*, has found, by
measuring from the *Tower of London* to the
Middle of the *City of York*, in the Year
1635, that one of those *Degrees*, upon the
Earth's Surface, contains $69\frac{1}{2}$ *Miles*; ac-
cording to which Measure, we find the
Earth's Circumference to be 25020 *Miles*—
its *Diameter* 7964—and its *Semidiameter*
3982.



Then in the Triangle ABC, Right Angled at B, we have the Side CB
= the *Earth's Semidiameter* 3982.—Also the Line AC = the *Semidiam-*
eter and Height of the Mountain together = 3986.—To find AB, the *Dis-*
tance from the Hill to the visible Horizon.

To Hypoth. AC 3986
Add Leg BC 3982

Sum 7968

Multiply by Differ. 4

Extract the Root 31872.00 (178.5 The Distance the
Mountain can be seen.

27)218

189

348)2972

2784

3565)18800

17825

975

This *Mountain* can be seen 178.5 *Miles* at *Sea*.

PROBLEM

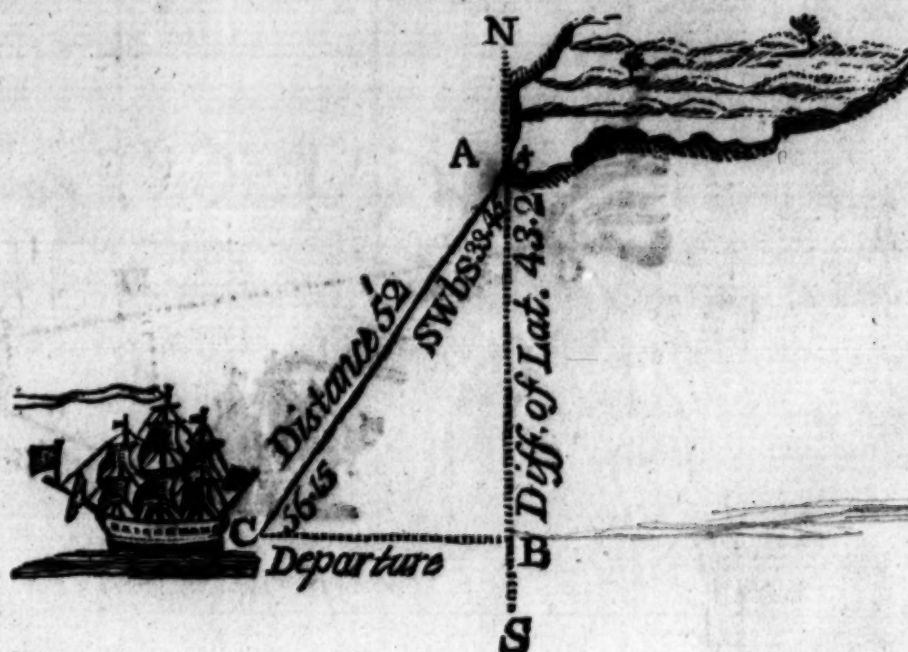
PROBLEM XI.

The *Distance run at Sea*, and the *Course*, given; to find the *Difference of Latitude* and *Departure* from the *Meridian*.

Suppose a *Ship* from A, in the *Latitude* of 50° North, fails away, SW by S. 52 Miles, to C: I demand the *Latitude* she is in, and also her *Departure* from the *Meridian*.

33.75
33.75
16875
23625
10125
10125
1139.0625
3
3.417.1875
57.3

Nat. Rad. 60.7 is enough.



(1st.) For the *Departure*,

N. Rad. Hyp. AC $\angle A$
As 60.7 — 52 — 33.75

6750
16875

60.7)1755.00(28.9 The *Departure*
1214

5410
4856

5540
5463

77

Latitude departed from 50° North
Difference of Latitude 0 43.2
Latitude the Ship is in 49 16.8

(2d.) For the *Diff. of Latitude* AB.

To Hypoth. AC 52
Add Side BC 28.9

Sum 80.9

Multiply by Differ. 23.1

809

2427

1618

Extract the Root 1868.79(43.2 *Diff. of Latitude*
16

83)268

249

862)1979

1724

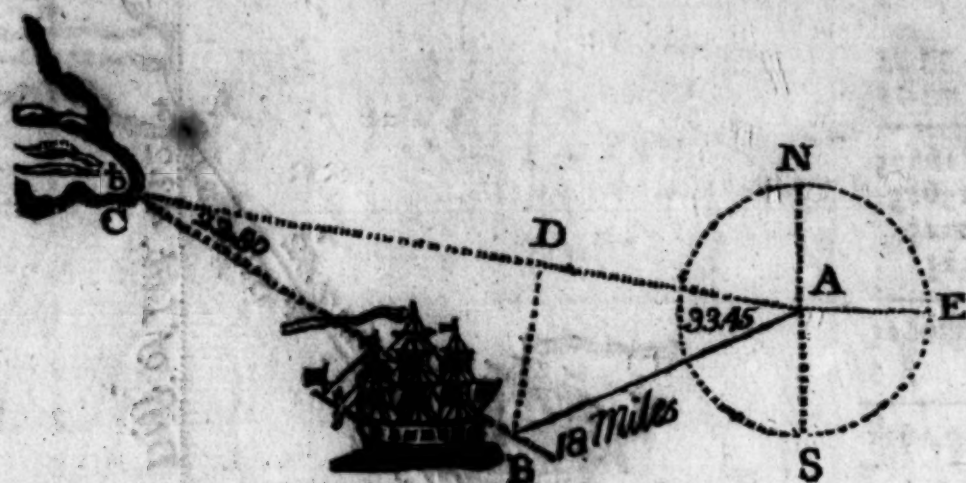
255

NOTE. That in all Cases of *Sailing*, we suppose the *Top* of the Book *North*, and *Bottom South*; the *Right Hand East*, *Left Hand West*.—The *Distance run* is the *Hypothenufe*; the *Difference of Latitude* is the *Perpendicular*; the *Departure* the *Base*. The *Angle* at the *Perpendicular* is the *Course*, and the *other* its *Complement*.

PROBLEM XII.

To take the *Distance* of any *Cape, Fort, or Island*, from a *Ship at Sea*.

Sailing W. S. W. I saw, at some Distance, a Point of Land, which I *set*, and find it bears from me W. by N. and having sailed 6 Leagues further, I find it then bears from me N. W. by W. I would know how far this Land is from me.



(1st.) Find the Perpend. BD in Triangle ABD.

$$\begin{array}{r}
 33.75 \\
 33.75 \\
 \hline
 16875 \\
 23625 \\
 10125 \\
 10125 \\
 \hline
 1139.0625 \\
 3 \\
 \hline
 3.417.1875 \\
 57.3
 \end{array}$$

Nat. Rad. 60.7 is enough.

N. Rad.	Op.	Side AB	∠ A
As 60.7	—	18	— 33.75
			18
			27000
			3375
			60.7)607.50(10 Perpend.
			607
			.50

(2d.) Find the Distance CB in Triangle BCD.

$$\begin{array}{r}
 22.5 \\
 22.5 \\
 \hline
 1125 \\
 450 \\
 450 \\
 \hline
 506.25 \\
 3 \\
 \hline
 1.518.75 \\
 57.3
 \end{array}$$

Nat. Rad. 58.8 is enough.

∠ C	Op.	Side BD	Nat. Rad.
As 22.5	—	10	— 58.8
			10
			22.5)5880(26.13
			450
			1380
			1350
			300
			225
			750
			675
			75

Answer, 26.13 Miles, the Distance requir'd.

PROBLEM

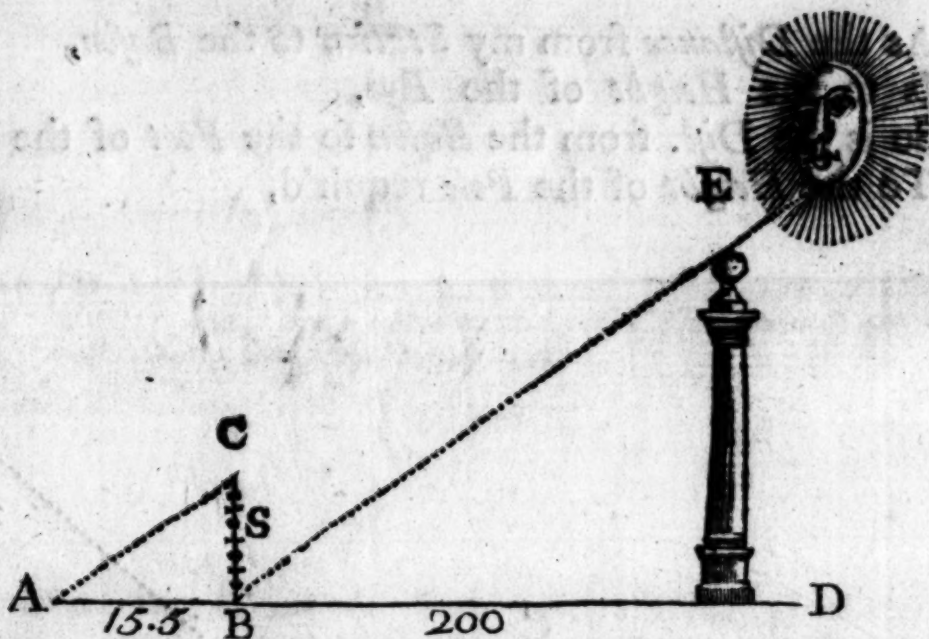
PROBLEM XIII.

To take the Height of a *Tree, Fort, Obelisk, Pyramid*, or any *Object*, by a *common Stick* only, when the *Sun* or *Moon* shines upon it.

Take a Stick of any Length, suppose 8 Feet; set it upright upon the Ground, as at CB in the Figure below. Mark the End of its Shadow at A, and measure its Length from B to A, which suppose to be 15.5 Feet. Then measure the Length of the Shadow of the *Pillar* or *Obelisk* BD, which suppose to be 200 Feet. This being done, you may easily find the Height: For (by Reason of like Triangles) it will always hold,—as the Length of the Shadow of the Stick AB in the small Triangle, is to its Height CB; so is the Length of the Shadow of the Obelisk BD in the great Triangle, to DE the Height thereof.

Shad. St. Stick. Shad. Ob.
As 15.5 — 8 — 200

15.5)1600.0(103.2
155 .. Height
— requir'd
.. 500
465
—
.. 350
310
—
.. 40



In this *Manner* the Heights of the *Pyramids* in *Egypt* have been taken. Those stupendous Buildings are supposed to have been erected by the *Children of Israel*, when in Bondage, for *Sepulchers* for the *Egyptian Kings*. They are the greatest Pieces of *Antiquity* now in Existence. There are several smaller Ones, but the largest, which is justly esteemed one of the *Wonders of the World*, is 500 Feet in *Perpendicular Height*;—700 Feet if measured *obliquely* from the Bottom to the Top;—and its *Base* covers about 11 Acres of Ground.

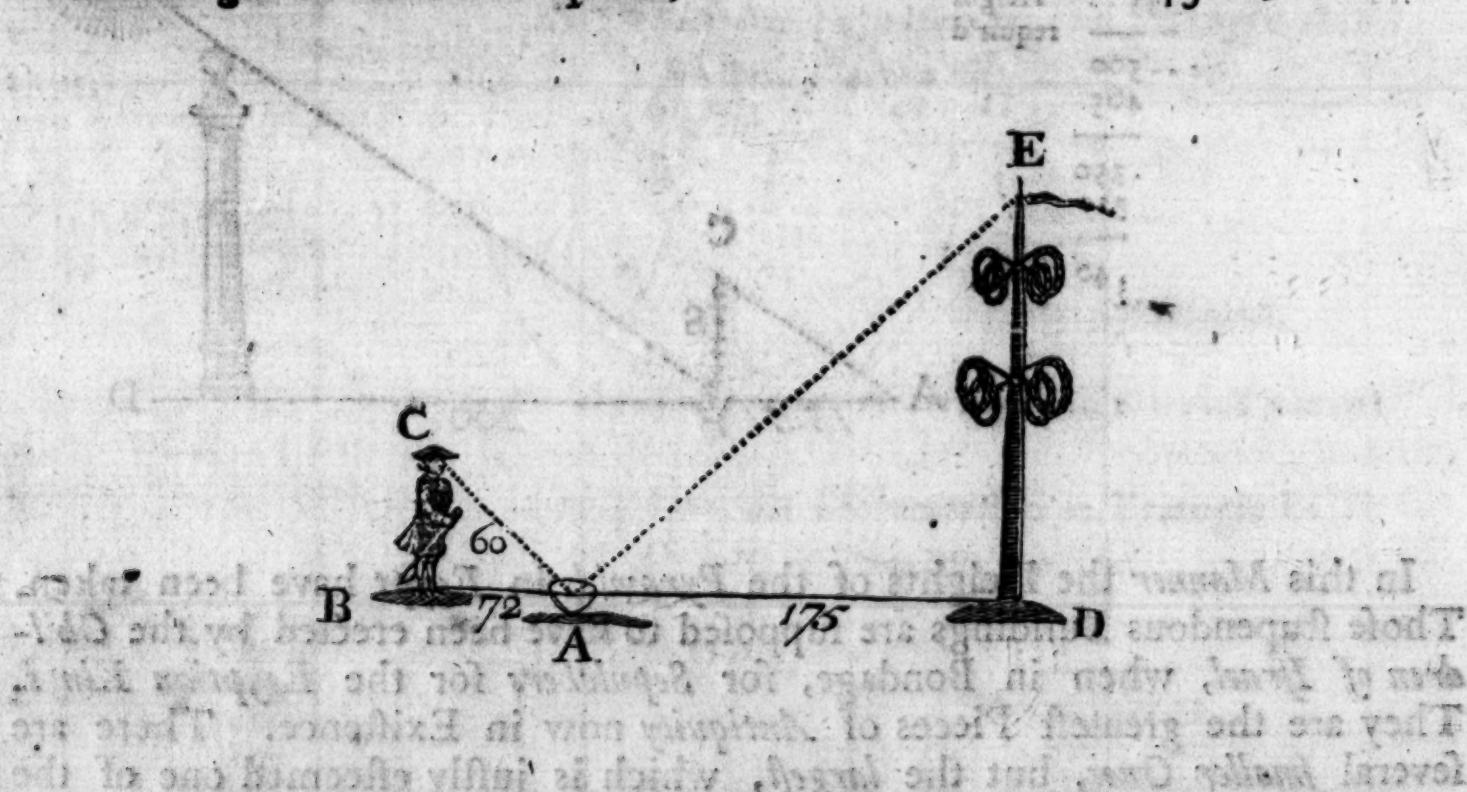
PROBLEM

PROBLEM XIV.

To take the *Height* of any *accessible Object* by a *Basin of Water*, or common *Looking-glass*.

Travelling along the Road, I see a fine *May-pole*, whose *Height* I would gladly know; but having no *Mathematical Instrument* with me, I procure a *Basin of Water*, which I set upon the Ground, at some *Distance* from the Pole, as at A; then I go backwards, till I see the *Top* of the Pole in the *Middle* of the Water, as at B. This done, I measure the *Distance* from my Station at B to the Basin at A, which suppose 72 Inches; and also measure from the Basin to the *Bottom* of the Pole at D, and find it 175 Inches. Next I measure the *Height* of the Eye from the Ground, which suppose 60 Inches. Then say, by the *Rule of Three*,

As the <i>Distance</i> from my <i>Station</i> to the <i>Basin</i> ,	= 72	} Inches.
Is to the <i>Height</i> of the <i>Eye</i> ,	= 60	
So is the <i>Dist.</i> from the <i>Basin</i> to the <i>Foot</i> of the <i>Pole</i> ,	= 175	
To the <i>Height</i> of the <i>Pole</i> requir'd,	= 145.8	



The same Thing may be obtain'd by a *Looking-glass*, laid truly *Horizontal*, or level on the Ground, by walking back till you can see the *Top* of the *Building*, &c. in the *Middle* of it, as was done by the *Water*.

PROBLEM

PROBLEM XV.

To take the Distance of the *Sun*, *Moon*, or any of the *Heavenly Bodies*.

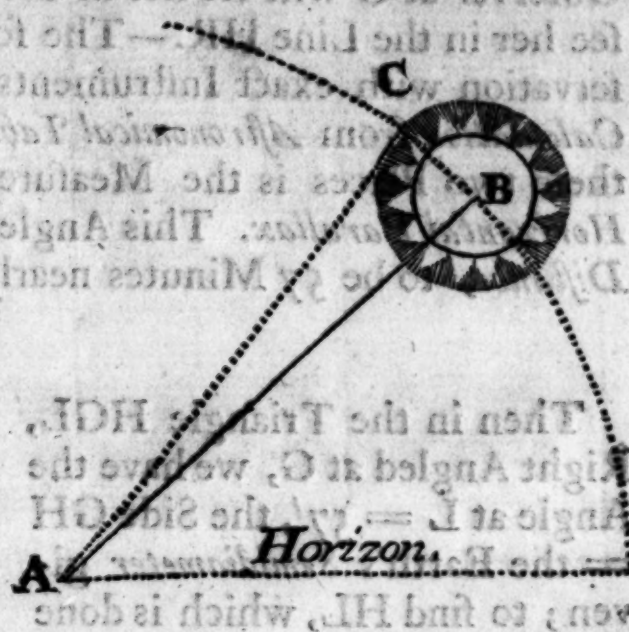
Suppose it was requir'd to find the Distance of the *Sun*, in *Diameters* of his Body from us.

With a *Quadrant* nicely graduated, take the *Altitude* of the lower and upper *Limb* in *Degrees* and *Minutes*, and subtract the one from the other; the Remainder will give the *Diameter* of the *Sun*, which we will suppose, in this Case, to be 32 Minutes.

Then have we given in the Triangle ABC, Right Angled at B,—the Angle at A = 16', and the Side CB = .5 = Half the Diameter of the Sun, whose whole Diameter we will call 1; to find the Distance, or Side AC.

Ang. A : Side BC :: Nat. Rad.
As .26 — .5 — 57.3

.26)2865(110 Diameters; and so far is the *Sun* of its own Breadths from us in the Winter*, but in Summer, the Angle being a little smaller, he must, consequently, be a little further from us.



Having found the *Distance* of any *Heavenly Body* in its own *Diameters* from us; you may easily tell its *Distance* in *Miles*, if you first know the *Diameter* of that Body in *Miles*. For, the Distance in *Diameters*, multiply'd by the *Miles* in one *Diameter*, gives the Distance sought.

For the Use of the Learner, I have here subjoin'd a Table of the *Diameters* of all the *Planets* in *English Miles*; whose *Distances* he may calculate at his Leisure.

Sun 800.000—Mercury 2460—Venus 7906—Earth 7964—Mars 4444
Jupiter 81.155—Saturn 67.870—Moon 2175.

* In this Manner we can tell the *apparent* Distance of any of the *Heavenly Bodies*: For the *Sun* appearing about 1 Foot in Diameter, his *apparent* Distance can be only 110 Feet, or 37 Yards. The *apparent* Distance of the *Moon* is nearly the same.

PROBLEM XVI.

To find the *Distance* of the *Moon* from the *Earth* another Way.

As the Places of all the *Heavenly Bodies* are computed from the *Center* of the *Earth*, but we are obliged to view them from the *Superfices*, they must therefore appear something lower in the *Heavens* than they really are.

Thus, suppose, for Instance, the ν to be at L in the visible Horizon; an Observer at G will see her in the Line GN, but an Eye in the Center will see her in the Line HR.—The former is her *apparent Place*, known by Observation with exact Instruments; the latter her *true Place*, and known by Calculation from *Astronomical Tables* of her Motion. The Difference between these two Places is the Measure of the Angle GLH, which is called the *Horizontal Parallax*. This Angle has been found, when the ν was at a *mean Distance*, to be 57 Minutes nearly.

Then in the Triangle HGL, Right Angled at G, we have the Angle at L = $57'$, the Side GH = the Earth's *Semidiameter* given; to find HL, which is done thus.

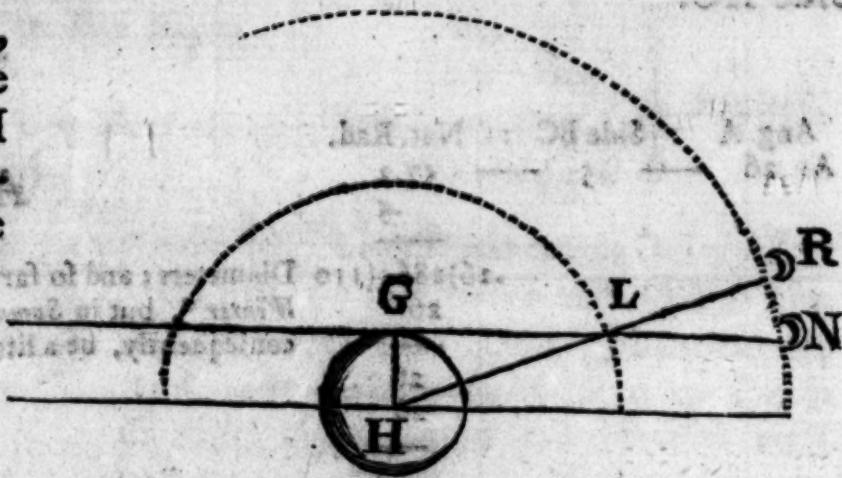
Ang L : Semid. GH :: N. Rad.
As .95 — 1 — 57.3

.95)57.30(60 Semidiameters : which multiply'd by the Earth's Semidiameter—4000 nearly, you have 240000 Miles the Distance of the Moon sought.

When the ν is at her *least* or *greatest* Distance from the Earth, she will be about 3 *Semidiameters* of the Earth *nearer* or *farther off*, than at her *mean Distance*.

In like Manner may the Distance of any of the *Planets*, *Comets*, or other *caelestial Phænomenon* be determined, by obtaining its *Parallax* in the *Horizon*.

NOTE. To find the *Distance* of any Place to which the *Sun*, *Moon*, or any *Star* is *vertical*, i. e. over its Head, or in its *Zenith*. See my *Geography*, p. 30.



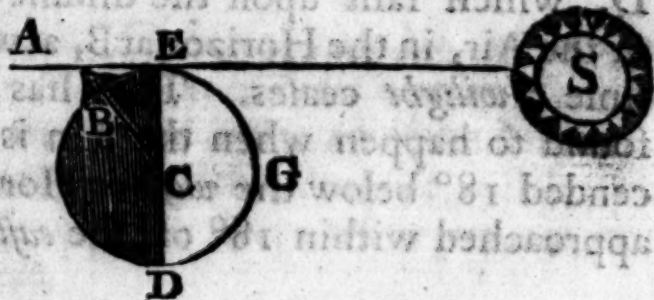
PROBLEM XVII.

To measure the *Height* of a *Mountain* in the *Moon*.

The *Moon* is suppos'd to be compos'd of *Land* and *Water* as our *Earth* is; consequently there must be some Unevennesses, or Inequalities, of *Hills* and *Vallies* as here. This indeed is confirmed by viewing her through a good *Telescope*; for then we find, that the Line, which separates the Light from the Dark Parts on her Surface is not even or regular, but tooth'd and jagg'd with innumerable Breaks; and even in the dark Parts, near the Borders of the lucid Surface, there are seen some small Spots enlighten'd by the Sun, which are very visible when the ☾ is three or four Days old, and which can be nothing else but the Tops of *Mountains* or *Rocks*; since it is impossible for the Sun's Rays to fall upon those Parts only, unless they were higher t' an the Rest of the Surface.

The *Lunar Mountains* are found to be *higher*, in Proportion to the Body of the ☾ than any Hills upon our Globe.—The Manner of calculating their Heights is this.

Let EGD be the Surface of the ☾ and ECD the Diameter of the Circle bounding *Light* and *Darkness*. A the Top of a Hill within the dark Part, when it first begins to be illuminated by a Ray of Light coming from the Sun at S. Then observe with a Telescope the Proportion of the Right Line AE, (*i. e.* the Distance of the Point A from the lucid Part) to the Diameter (or Semidiameter) of the ☾ ED, for that being ascertain'd, you have in the Triangle AEC, Right Angled at E, (where the Ray of Light touches the ☾) the two Sides AE and CE, to find the *Hypotenuse* AC, from which subtracting BC = EC, there will remain AB the Height of the *Mountain*.



Ricciolus, on viewing the ☾ when about four Days old, observ'd the Top of a Hill called *Saint Catherine*, near the N. Part of *Mount Taurus*, (see my *Astronomy*) to be illuminated, and that it was then distant from the Surface about $\frac{1}{4}$ of the Moon's *Semidiameter*. Now as the *Semidiameter* of the ☾ EC is about 1088 Miles, the Line AE being $\frac{1}{4}$ of it, must be = 136 Miles. Consequently, if the $\square$ of EC and $\square$ of AE be added together, and then the $\square$ Root of it be extracted, it will give the Line AC, from which subtracting the Moon's *Semidiameter* BC or CE, the Remainder, which is 8 Miles will be the Height of the *Mountain* sought.

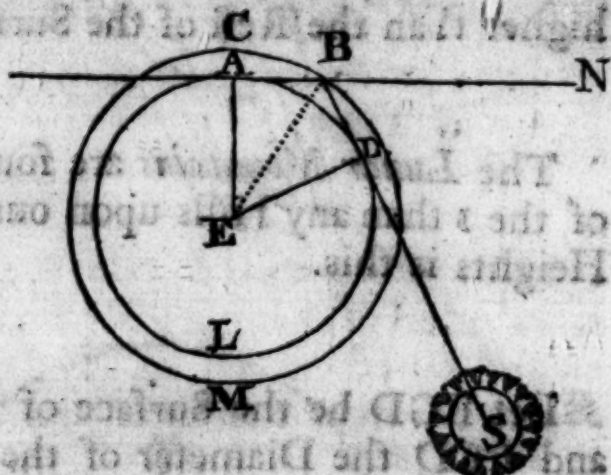
PROBLEM

PROBLEM XVIII.

To measure the *Height* of the *Atmosphere*.

The *Atmosphere* is that Circle of *vaporous Air* furrounding the Earth, which being illuminated by the *Sun's Rays* makes the Brightness and Glory of the *Firmament* we behold, whilst the Sun continues above the *Horizon*. And, after the Sun is gone down, the *Atmosphere*, which is higher than we are, will still continue to be illuminated by those Rays passing by the Earth over our Heads; but this *Brightness* grows *less* and *less* as the Sun descends lower, till he arrives at 18° below the *Horizon*; when all the Parts of the Air above fall out of his Rays, and, consequently, become dark.

To make this plainer; suppose the inner Circle ADL represents the Earth, and the Circle CBM the *Atmosphere*. Suppose a Person standing upon the Earth at A whose *sensible Horizon* is AN. Also let SB be a Ray of Light coming from the *Sun*, touching the Earth at D, which falls upon the distant Part of the Air, in the *Horizon* at B, at which time *Twilight* ceases. This has been found to happen when the Sun is descended 18° below the *western Horizon* in the *Evening*; and also when he is approached within 18° of the *eastern Horizon* in the *Morning*.



Now as the Arch AD is 18° , we have, by drawing the Line EB, two equal Triangles, from either of which we may find the Height of the *Atmosphere* requir'd. — For in the Triangle ABE, Right Angled at A, we have given the Side AE the *Earth's Semidiameter*, and the Angle AEB = 9° = half the Arch of the *Sun's Descent* below the *Horizon*, and the Angle ABE = 81° , to find the *Hypothenuse* EB, from which if you subtract the *Semidiameter* of the Earth, the Remainder will be the Height of the *Atmosphere*.

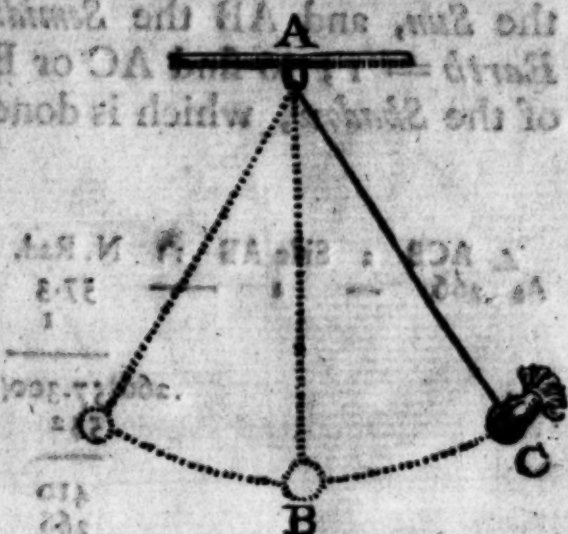
Thus, Ang. B : Earth's Semid. AE :: N. Rad. : Hypoth. EB
 As 81° ————— 4000 ————— 81.99 ————— 4048
 4000 Earth's Semidiameter subtract
 48 Height of the Atmosphere requir'd.

PROBLEM

PROBLEMO XIX.

To measure the *Distance* of a *Cloud*, from which issues *Lightnings* and *Thunder*.

Take a small *Ball* of *Lead*, *Ivory*, or any other matter, and affix it to the End of a fine *Thread*. Then measure from the Center of the *Ball*, along the *Thread*, exactly 39.2 *Inches*, where make a *Loop*. This done, suspend it by that *Loop* to the *Ceiling* of the *Room*, or to any other *Place* where it may hang freely, and vibrate backwards and forwards like a *Pendulum*, as in this *Figure*. Now the Property of this little *Instrument* is, that each *Vibration*, whether it passes through a larger or a smaller *Space*, will be performed in one *Second* of *Time*.



Being thus prepar'd, take the *Ball* in your *Hand*, and drawing it aside from its perpendicular *Direction* *AB*, to any *Distance*, suppose to *C*, hold it there till you see the flash of *Lightning* pass by, at which *Moment* let it go, and count the *Number* of *Vibrations* till you hear the *Stroke* of the *Thunder*. Then these *Vibrations* multiplied by 1142, (the *Number* of *Feet*, *Sound* uniformly passes thro' in each *Second*) the *Product* will be the *Height* of the *Cloud* in *Feet*, if it be nearly over the *Place* where you are; or its *Distance* from you in any other *Situation*.

Thus, suppose the *String* is found to make 8 *Vibrations* between the *Lightning* and the *Thunder*; then $8 \times 1142 = 9136$ *Feet*, which, divided by 5280 (the *Feet* in 1 *Mile*) gives 1 $\frac{1}{2}$ *Mile* nearly; and so far is that alarming *Tempest* from you.

In this *Manner* you may continue to measure the *Distance* of the *Cloud* all the *Time* it passes from your *Zenith* to the *Horizon*, and by that *Means* be acquainted with the *Danger* it seems to threaten the *Neighbourhood*, as well as the *Extent* of the visible *Hemisphere* of *Clouds*.

The *Distance* also of a *Ship* at *Sea*, or a *Fort*, may be estimated in the same *Manner*, by counting the *Vibrations* from the *Flash* of the *Powder* to the *Report* of the *Gun*.

PROBLEM XX.

To calculate the Length of the Earth's or Moon's Shadow.

The Angle of the Cone ACD of the Earth's Shadow in the annex'd Figure, is equal to the Sun's apparent Diameter*, which, at a mean Distance from us, is about 32 Minutes.—Hence, in the Triangle ACB, Right Angled at B, we have the Angle ACB = 16 Minutes, the apparent Semidiameter of the Sun, and AB the Semidiameter of the Earth = 1; to find AC or BC the Length of the Shadow, which is done thus.

$\angle$ ACB : Side AB :: N. Rad.
 As .266 — 1 — 57.3

.266)57.300(215 Semidiameters = AC

532

410

266

1440

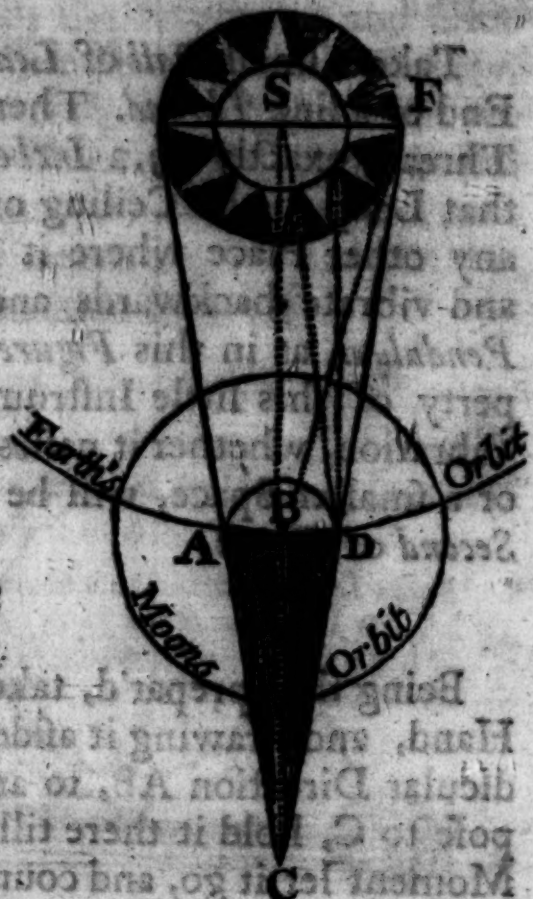
1330

110

Thus, when the Sun is at a mean Distance from us, the Shadow of the Earth reaches about 215 Semidiameters beyond it: But when the Sun is at his greatest or least Distance, the Shadow will be lengthened or shortened 3 or 4 Semidiameters, more or less.

Hence, you may also determine the Height of the Moon's Shadow: For, as the Moon is never at any great Distance from the Earth, the apparent Semidiameter of the Sun must be nearly the same there as here. Consequently, the Moon's Shadow must contain the same Number of Semidiameters of the Moon, as the Earth's Shadow does Semidiameters of the Earth: Which Semidiameters, multiply'd by the Miles in the Semidiameter of the Moon or Earth, will give the Length of the Shadow respectively in Miles.

* The Semiangle of the Cone of the Earth's Shadow BCD, is equal to the apparent Semidiameter of the Sun view'd from the Top of the Shadow, which Angle is always equal (in the Shadow of every Planet) to the apparent Semidiameter of the Sun SBF, less'n'd by his Horizontal Parallax BSD at that Planet. But as the Horizontal Parallax of the Sun, i. e. the Angle under which the Earth is seen from thence, is scarcely 10 Seconds, it may be omitted, as is done in the above Calculation.



PROBLEM

PROBLEM XXI.

To calculate the *Diameter* of the *Earth's Shadow* at the *Distance* of the *Moon*; and also, the *Diameter* of the *Moon's Shadow* at the *Earth*.

In the following Figure let *S* represent the *Sun*, *E* the *Center* of the *Earth*, *M* the *Moon*, *EC* the *Cone* of the *Earth's Shadow* (at a mean) = 215 *Semidiameters* of the *Earth*: Then *MC* will be the *Cone* of the *Earth's Shadow* reaching beyond the *Moon*, whose *Length* is thus found.

From *EC* the *Cone* of the *Earth's Shadow* = 215
Subtract *EM* the *Dist.* of the *Moon* in *Earth's Semid.* = 60
Remains *MC* the *Shad.* of the *Earth* beyond the *Moon* = 155

Then, by Reason of similar Triangles, it will always hold;

As the *Length* of the whole *Shadow* Is to the *Diameter* of the *Earth* So is the *Length* of the *Shadow* beyond the *Moon* To the *Diameter* of the *Shadow* at the *Moon*

EC ab MC
As 215 7964 155
 155
 39820
 39820
 7964

215)1234420(5741 Miles = *cd*, the *Diameter* of the *Earth's Shadow* at the *Distance* of the *Moon*.

1075

1594
1505

892

860

320

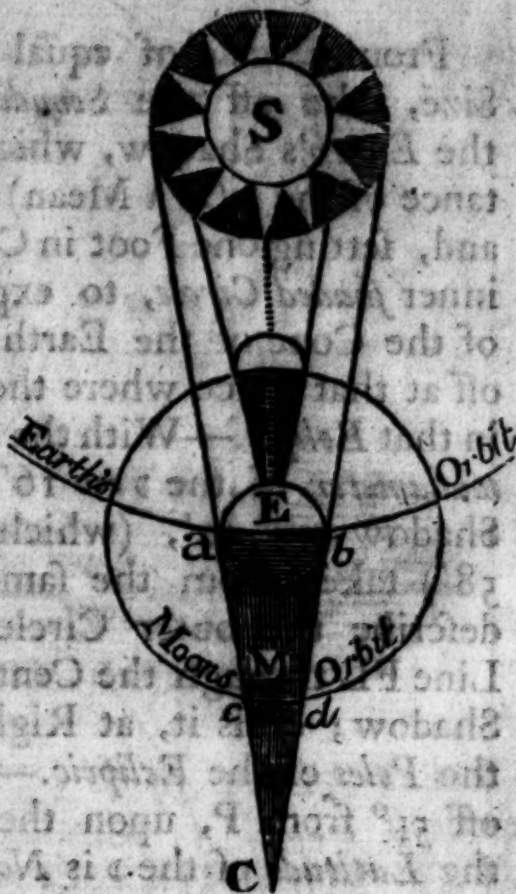
215

105

By this Problem also may be found the *Diameter* of the *Moon's Shadow* at the *Surface* of the *Earth*, and, consequently, how much of the *Earth* is involv'd in that *Shadow* in an *Eclipse* of the *Sun*. For the *Length* of the *Moon's Shadow* is found to be about 60 *Semidiameters* of the *Earth*, which is nearly the *Moon's mean* *Distance* from us; her *Shadow*, in that State, must, therefore, reach as far as the *Center* of the *Earth*.—But as the *Moon* is, sometimes, almost 4 *Semidiameters* of the *Earth* nearer, the *Shadow* must reach 4 *Semidiameters* beyond the *Center* of the *Earth*; and when the *Moon* (as she sometimes is) is 4 *Semidiameters* further from us, the *Shadow* will then not reach the *Earth* at all. In such Case, the *Sun*, though centrally eclips'd, will not be totally covered by the *Moon*; but an *Annulus*, or Ring of Light, will appear round the *Border* of that *Luminary*, as happen'd April 1st, 1764.

* The Method of finding the *Distance* of the *Moon*, in *Semidiameters* of the *Earth*, is shewn at Problem XVI.

PROBLEM

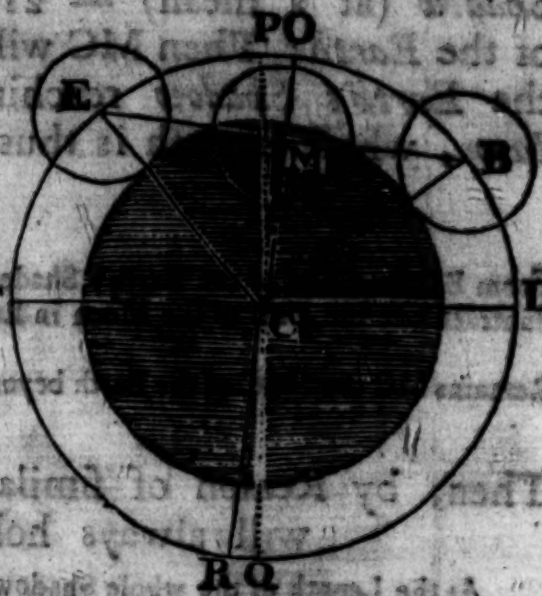


PROBLEM XXII.

To calculate the Beginning, End, and total Duration of an Eclipse.

Having, from *Astronomical Tables*, obtain'd the Time of the Middle of the Eclipse, which suppose to be December 21 Day, 11 Hours, 49 Minutes, with the Latitude of the Moon at that Time, = 40 Minutes, you may then proceed to find the Beginning, End, and total Duration, as follows.

From a Scale of equal Parts, of any Size, take off the Semidiameter of the the Earth's Shadow, which, at the Distance of the ☾ (at a Mean) is about 42'; and, setting one Foot in C, describe the inner shaded Circle, to express that Part of the Cone of the Earth's Shadow cut off at that Place where the ☾ passes thro' in that Eclipse.—With the Sum of the Semidiameter of the ☾ = 16', and Earth's Shadow = 42', (which together = 58') taken from the same equal Parts describe the outer Circle.—Draw the Line FL through the Center, to represent the *Ecliptic*, or Path of the Earth's Shadow; cross it, at Right Angles, with the dotted Line PQ, to express the Poles of the *Ecliptic*.—Then with a Line of Chords, or a Protractor, set off $5\frac{1}{2}^\circ$ from P, upon the outer Circle, towards the Right Hand, because the Latitude of the ☾ is North ascending, to express the Angle of the Moon's Path with the *Ecliptic*, and draw the Line OR.—Take the Latitude of the ☾ = 40', from the same Scale of equal Parts, and set it from C to M upon the Line CO.—Then draw a Line through M, at Right Angles to CO, and that Line will represent the Path of the ☾ during the Eclipse.—Next, with the Semidiameter of the ☾ = 16', taken from the equal Parts, describe, on the three Points B, M, and E, severally, the three little Circles; so will the Circle at B represent the ☾ at the Beginning, that at M the Middle, and that at E, the End of the Eclipse.



Now, from the Center C, draw two Lines to B and E; then in the Right Angled Triangle CMB, Right Angled at M, we have given CM the Latitude of the ☾ = 40, and CB = CE the Sum of the Semidiameters of the Moon and and Earth's Shadow = 58; to find MB = ME, the Motion of Half Duration of the Eclipse.

From Square of 58 = 3364
Take Square of 40 = 1600

Extract the Root

1764 (42 Minutes, = the Motion of Half the Duration. And because the Moon passes over 31 of these Minutes (at a mean Rate) in one Hour, we have this Proportion.—If 31 Min. :: 1 Hour :: 42 Min. :: 1 Hour 21 Min. which subtracted from, and added to, the Middle, will give the Beginning and End.

D. H. M.
Middle of the Eclipse Dec. 21 11 49
Half Duration subtract and add 1 21

Beginning 21 10 28
End 21 13 10

Total Duration 2 42

NOTE. The Middle of an Eclipse, with the Lat. of the Moon, may be easily had from my *Astronomy*.

PROBLEM

PROBLEM XXIII.

To take the *Distance* of an *inaccessible Object* without the Help of any *Instrument*.

Suppose E, in the following Figure, to be a *Fort*, whose *Distance* you want to know, and you cannot approach it, on Account of some *Moat*, *Ditch*, or *River*, lying between you and the *Object*.



First, at some *Distance* from the *Ditch* or *River*, set up a *Stick*, as at C; then advance, in a *Right Line*, towards E, any Number of Yards, suppose 50, and set up another *Stick* at A; next, move, in a *Line perpendicular* to CE, from A to B, any *Distance*, suppose 66 Yards, and set up another *Stick* at B; then return back to C, where you began, and remove from thence, in a *Line perpendicular* to CE, till you see the *Stick* at B and the *Object* E in a *Right Line*, and set up another *Stick* in that Place at D, measuring the *Distance* from C to D, which suppose 76 Yards. Then it will always hold;—

As the Difference between AB and CD	= 10	} Yards.
Is to the Length between A and C	= 50	
So is the Distance CD	= 76	
To the Distance between C and E	= 380	

NOTE. If, in the *third Term*, you had us'd the *Distance* AB = 66 Yards, you would have obtain'd the *Distance* from A to E = 330.

PROBLEM

PROBLEM XXIV.

To find, by a *new Method*, where the *Bees* hive in large and extensive *Woods*, in Order to obtain their *Honey*.

Take a *Plate* or small Piece of *Board*, on which is spread a little *Honey* or *Treacle*, and set it down on a *Rock* or *Stump* of a *Tree* within the *Wood*. This the *Bees* will soon find out if any are near, for it is generally believ'd they smell Things of that Nature at the Distance of a *Mile*, or further. Whilst these little *Creatures* are feeding, secure *two* or *three* of them in a *Box*, or something convenient. Then let one of them go, observing carefully, by a *Pocket Compass*, the Course he takes; for *Bees*, after they rise in the *Air*, fly directly in a strait Line to the *Tree* where their *Hive* is.

Suppose, for Example, the *first Bee* is found to fly directly *West*; then you may be sure the *Tree* is some where in that Line from your present Station. But, in order to know how far, you must make an *Offset*, either *North* or *South*, as large as you can, which in this case we will suppose to be 100 *Rods* or *Perches* (the larger the better) to the *South*. Here you must let go another *Bee*, observing his Course as before, (for this *Bee*, being loaded like the other, will fly directly to the *Hive*) which Course we will suppose to be N. W. by W. — $56^{\circ} 15'$ towards the *West*, it only remains now to find where these two Courses or Lines intersect or meet with each other, for there you will find the *Tree* in which the *Honey* is.

This may be easily done.—For in the *Right Angled Triangle* ABC, are given the *Right Angle* B, the *Course* of the first *Bee*, the *Angle* at A, the *Course* of the second *Bee*, and the *Distance* AB; to find BC, or AC, the *Distance* of the *Tree* from either Station.

$$\angle C : \text{Base} :: \text{N. Rad.} : \text{Dist. AC}$$

$$\text{As } 33.75 - 100 - 60.7 - 179.8 \text{ Perches.}$$



Formerly, they found the *Honey* by surprizing the *Bees*, and following them, one after another, till they found out the *Hive*; but since this *Trigonometrical Method* has been us'd, the *Searchers* discover that *Booty* in a few *Hours*, which before requir'd many *Days*.

CONCLUSION.



CONCLUSION.

THES E few *Problems* are sufficient to point out the great *Use* of this Branch of *Learning*. The Advantages resulting from it to Society are very great;—almost infinite.—Nothing however posited in the *Heavens*;—nothing upon the *Earth* or *Seas*;—but its *Distance* and *Dimensions* may be ascertained by it.—It is no Wonder then, that *Pythagoras*, a learned Philosopher of *Samos*, when he had discover'd that famous *Proposition* (47th of 1st Book of *Euclid*) which is the *Foundation* of this *Science*, should, in Gratitude, sacrifice an *Hecatomb*, i. e. 100 Oxen, to the *Muses*, for inspiring him with such an useful Invention, which he judg'd beyond the Power of human Abilities to discover.

Thus by one plain *Geometrical Figure*, having three Sides and three Angles, and assisted by the *Rule of Three*, you see what amazing *Truths* may be discover'd. This illustrates not only the old Motto,—*Tria sunt omnia*—but also proves the *Truth* of Mine in the *Title-page*.

*Cuncta Trigonus habet, satagit quæ docta Mathesis,
Ille aperit clausum quicquid Olympus habet.*

Which may be English'd thus;

*In Heaven the latent Science lay conceal'd
Till the Triangle came, and Truth reveal'd.*

F I N I S.

NOTICE

Just Publish'd, by the same **AUTHOR**.

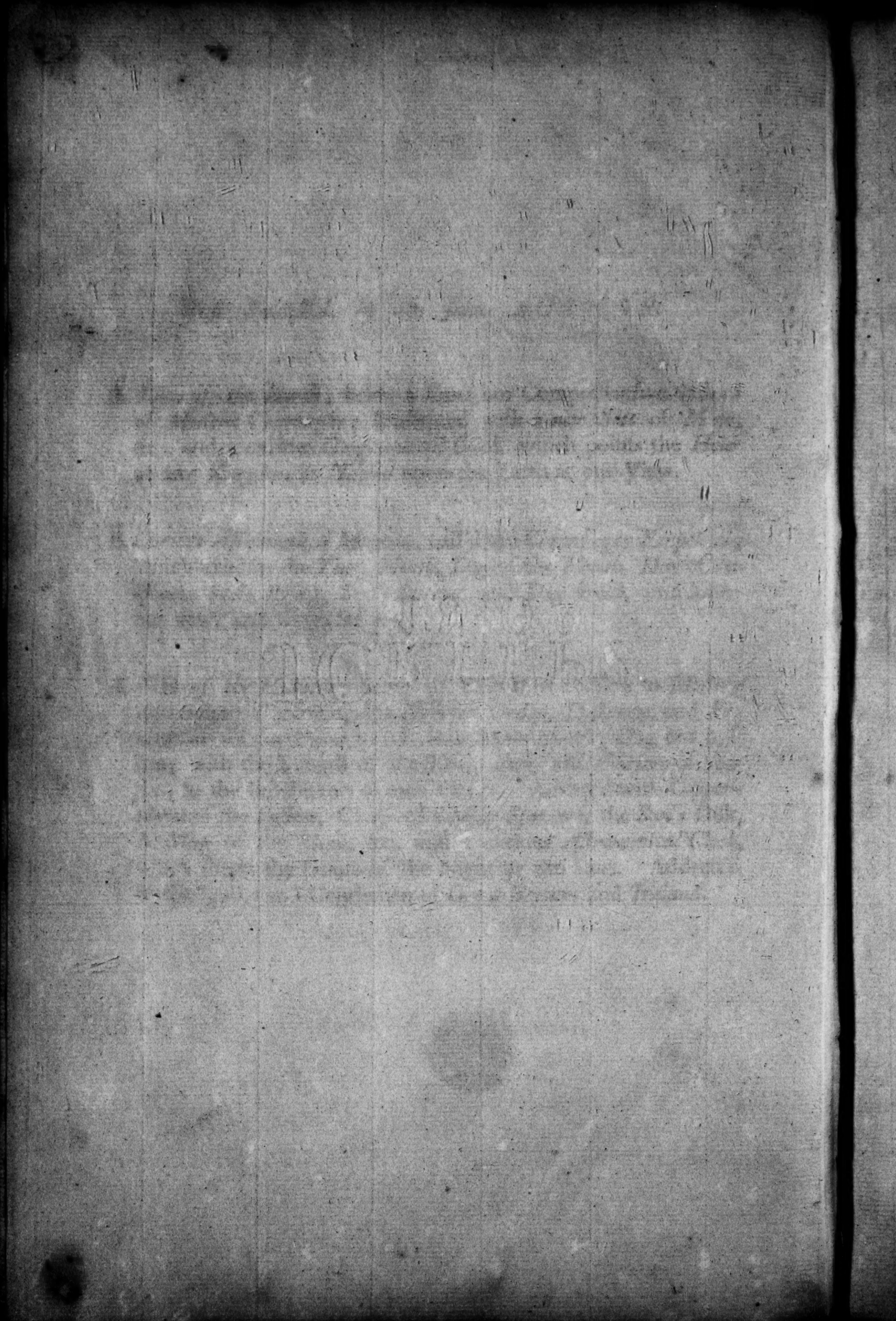
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